A DECISION SUPPORT SYSTEM FOR TOLERANCE ALLOCATION

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ABSTRACT

Tolerance design for discrete parts requires extensive data, handbooks and/or complex mathematical models. Decision support expert systems are needed that integrate such information to simplify tolerance design. In this research, user-friendly decision support tools for automated tolerance allocation and tolerance analysis are developed. The tolerance allocation prototype can provide component tolerances based on assembly specification. Handbook-based tolerances or optimal tolerances can be designed based on user interaction. Tolerance analysis determines the assembly tolerance for a given chain of component tolerances based on different analysis models and a decision-chart. Other research conducted include a methodology for indirect tolerance transfer to manufacturing and a new methodology for simultaneous process selection and tolerance allocation.

1. INTRODUCTION

Tolerances are fundamental to both design and manufacturing. Perfect parts are difficult to manufacture in large quantities. Tolerances are the allowable variation specified on the size, form and geometry of parts. Tolerance design is an experience-based discipline involving precision and cost considerations. A loose (large) tolerance invariably leads to a loss in the functionality while a tight tolerance (small zone) becomes very expensive. In assemblies involving fits (shaft in bushing, key in key slot, etc.), handbooks based on standards such as ANSI and ISO may be employed to calculate size tolerance zones for mating parts. These standards do not explicitly consider the cost of manufacture. Optimal tolerance allocation involves the minimization of the manufacturing cost subject to constraints on the assembly. The optimal allocation or distribution is used to find component tolerances for given assembly tolerances. The procedure for determination of the assembly tolerance based on component tolerances is termed as tolerance analysis. Over the last two and a half decades, several procedures have been suggested for tolerance allocation and tolerance analysis. This research sets out to create a decision support system for tolerance allocation and analysis that combines several of these techniques from literature within an integrated user-friendly framework.

In design tolerance allocation, the system queries the user for the type and class of fit and the size dimension. A database system created using ANSI standards is then used to retrieve tolerances for the appropriate dimensions and fits. Detailed information about the types and classes of fits is supplied to assist the user in making selections. The optimal tolerance allocation procedure is chained to this system and uses the design tolerance guidelines to retrieve cost parameters for optimization. An (linear or non-linear programming) optimization is conducted for minimization of cost (objective function) subject to assembly relationship (constraint), as per user selection. Optimal tolerances are displayed to user for each assembly chain selected, radial or linear. The tolerance analysis system supplies information on types of distributions and associated parameters to the user and requests data on component tolerances. A decision support system determines the most appropriate of worst-case, statistical, modified statistical, Spott's, mean-shift and moment's methods to calculate the assembly tolerance. The allocation as well as the analysis procedures are integrated within an expert system. In summary, a modular and flexible package is developed for dimensional (size) tolerance design for radial and linear dimensions of concentric rotational discrete parts.

Initially, an automated drafting procedure was integrated with the above expert systems. However, this system was not fully usable owing to the inflexibility of the drafting procedure and due to certain other programming constraints. Other research conducted includes the development of a methodology for indirect tolerance interpretation and transfer to manufacturing and a methodology based on advanced optimization methods for simultaneous process selection and tolerance allocation. The new method developed for process selection and tolerance allocation is simple, efficient and performs better (faster and more accurate) than the existing methods. The methodology is primarily developed for simple tolerance chains, but has been extended for simple interrelated tolerance chains.

The second chapter provides a detailed discussion of important papers in the areas of tolerance allocation, analysis and representation. The third chapter provides details on the prototype decision support systems created for tolerance allocation and analysis. The fourth chapter provides guidelines for the use of the software (an example is also provided). Chapter five provides a synopsis of the research conducted and prototype developed for tolerance interpretation and transfer. Process selection with tolerance specification for simple tolerance chains and interrelated chains is discussed in chapter six. Conclusions are detailed in chapter seven.

2. RELEVANT WORK IN TOLERANCING

A comprehensive literature review is presented in the next few sections describing the state of research in tolerance optimization, statistical tolerancing, and CAD representation of tolerances. Some other problems which are closely related to computer aided tolerancing are also discussed. Since the literature in each of these areas is closely related, it is quite difficult to separately classify them. However, an attempt is made to pool common topics together to attain the best possible results. The reader is urged to read the entire survey for a proper understanding of the topics in question.

The literature review in CAD tolerancing covers the following areas (among others):

(1) Problems in representing tolerances in conventional CAD modeling systems.

(2) Computer methods based on statistical and optimal tolerance allocation and analysis procedures.

(3) Techniques using knowledge based systems for tolerance allocation.

(4) Problems involving the selection of fits and allowances.

This survey is designed to help the reader get an overall idea of the problems discussed by various authors in their work in the tolerancing area. Most of the discussions included here may or may not be directly related to our research. However, the papers referred to have provided an insight into the present problem.

2.1 Computer Aided Tolerancing and CAD Tolerancing

Among the chief approaches used in tolerance literature, the ones in connection with CAD systems are gaining the highest importance. CAD systems (Kalpakjian, 1988) were developed to analytically plan the design of physical systems. Among the several advantages of analytical modeling is the ability to avoid prototyping and physical model testing. Two areas of major interest for CAD systems are: Numerical Control (NC) path planning and tolerance representation (Requicha and Voelcker, 1982). NC path planning is not relevant to this research and, hence, will not be covered in this review. Tolerance representation in CAD systems holds higher pertinence to our project. The inability to represent tolerance information within CAD systems makes tolerance representation very important. Tolerance allocation, selection of fits, and CAD based process planning are all related to tolerance representation.

By means of "interchangeable manufacturing", parts can be made in widely separated localities and brought together for assembly. Without interchangeable manufacturing, modern industry cannot exist. Simultaneously, without effective size control by the engineer, interchangeable manufacturing could not be achieved.

On the other hand, it is impossible to make anything to perfect size. Parts made to very tight dimensions are extremely expensive. However, perfect sizes are seldom needed for interchangability. A varying degree of accuracy may be permissible according to functional requirements. Hence, there is a need for a means to specify dimensions with whatever the required degree of accuracy and this accuracy could be achieved by specification of a tolerance on each dimension.

2.1.1 What is tolerancing?

Tolerance is the total amount a specific dimension is permitted to vary, i.e., it is the difference between the maximum and minimum limits (of variation). For example, a dimension given as 2.825 ± 0.002 means that it may be 2.827" or 2.823", or anywhere between these limit dimensions. The tolerance or total amount of variation is 0.004" (Giesecke et al, 1986).

Seven terms and definitions originate within the context of tolerancing: nominal size, basic size or dimension, limits, maximum material condition, least material condition, allowance, and fit. Giesecke et al (1986) define the terms as given below.

Nominal Size:

> "This is the designation which is used for the purpose of general identification of the dimension."

Basic Size or Dimension:

> "This is the theoretical size from which limits of size are derived by the application of allowances and tolerances."

Limits:

> "The maximum and minimum sizes indicated by a toleranced dimension."

Maximum Material Condition (MMC):

> "MMC means that a feature of a finished product contains the maximum amount of material permitted by toleranced size dimensions for that feature. Thus, we have MMC when internal features (holes, slots, etc.) are at their minimum size or when external features are at their maximum size."

Least Material Condition (LMC):

> "LMC means that a feature of a finished product contains the minimum amount of material permitted by toleranced size dimensions for that feature. Thus, we have LMC when internal features are at their maximum size or when external features are at their minimum size."

Allowance:

> "This is the minimum clearance space intended between the MMC of mating parts. So, allowance represents the tightest permissible fit and is simply the smallest hole minus the largest shaft."

Fit:

"Fit is the general term used to signify the range of tightness or looseness which may result from a specific combination of allowances and tolerances in mating parts. There are four basic types of fits: clearance fit, interference fit, transition fit, and line fit."

Tolerances are specified in several ways in engineering drawings (Giesecke et al, 1986). Giesecke et al (1986) describe some of the more common ways below:

(1) Limit Dimensioning:

"The maximum and minimum limits of size and location are specified as shown in Figure 2.1."

(2) Plus and Minus Dimensioning:

"The basic size is followed by a plus and minus expression of tolerance resulting in either unilateral or bilateral tolerances as shown in Figure 2.2. The unilateral system of tolerances allows variations in only one direction from the basic size. So, a unilateral tolerance is always all plus or all minus. The bilateral system of tolerances allows variation in both directions from the basic size as shown in Figure 2.2(b). Hence, there is a + specification. Plus and Minus Dimensioning is the most commonly used method to specify tolerances and will be used in this research."

(3) Single Limit Dimensioning:

"It is not always necessary to specify both limits. Either MIN or MAX is often placed after a number to indicate the minimum or maximum dimension desired. The other elements of design determine the other unspecified limit. For example, 0.25 MAX."

(4) Angular Tolerances:

"The specified angular tolerances are usually bilateral and are expressed in terms of degrees, minutes and seconds."

According to Requicha (1983), the current tolerancing standards and practices pose a representation problem in geometric modeling systems. Accordingly, a suggestion is made to tighten the current tolerancing standards and practices to help in representing them in computer-based geometric modeling systems in a form suitable for automatic planning of manufacturing.

At this point, some important concepts of geometric tolerancing such as size, form and position tolerances will be discussed in sections 2.1.2, 2.1.3, and 2.1.4.

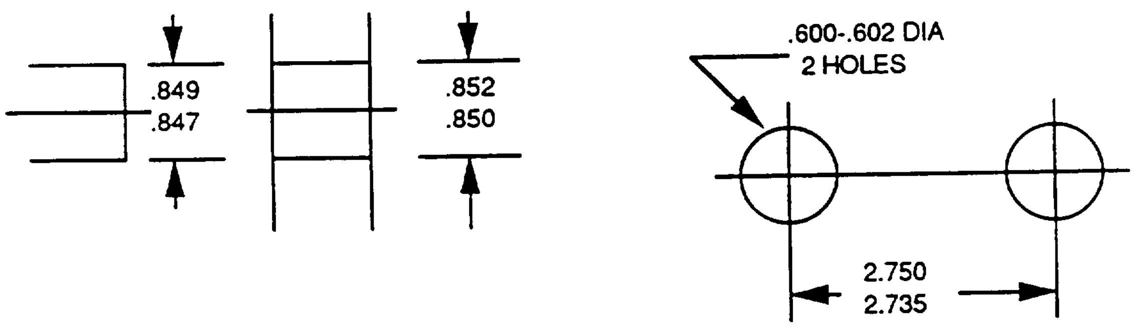


Figure 2.1. Limit dimensioning (Giesecke et al, 1986).

Figure 2.2. Plus and minus dimensioning (Giesecke et al, 1986).

2.1.2 Geometric Tolerancing

The term geometric tolerance refers tie the maximum allowable variation of a form or its position from the perfect geometry implied on the drawing. Also, the term "geometric" simply refers to various forms such as a plane, a cylinder, a cone, a square, etc. Since it is impossible to produce a perfect form, some amount of permissible variation must be specified. Thus the geometric tolerances specify either the diameter or the width of a tolerance zone--within which a surface or the axis of a cylinder must lie if the part is to meet the required accuracy for proper function and fit. For example, a geometric (form) tolerance may refer to a feature's straightness, parallelism, flatness, perpendicularity, and roundness.

2.1.3 Positional Tolerances

Positional tolerances deal with the permissible variations in locating a feature (or a part) with respect to some datum (reference) feature/part. Consider, for example, a hole located from two surfaces at right angles to each other as shown in Figure 2.3(a). As shown in Figure 2.3(b), the center may be within a square tolerance zone. The sides of this zone are equal to the tolerances. But the total variation along the diagonal of the square (0.014) will be $\sqrt{2}$ (diagonal of a unit square) times greater than the indicated tolerance (0.010).

Now, consider four holes dimensioned with rectangular coordinates as shown in Figure 2.4(a). Acceptable patterns of square tolerance zones are shown in Figure 2.4(b) and Figure 2.4(c). The positional (locational) tolerances which represent geometric characteristics such as concentricity and position are actually greater than indicated by the dimensions. The square tolerance zone for hole *A* results from the tolerances on the two rectangular coordinate dimensions locating hole *A*. The sizes of the tolerance zones for the other three holes result from

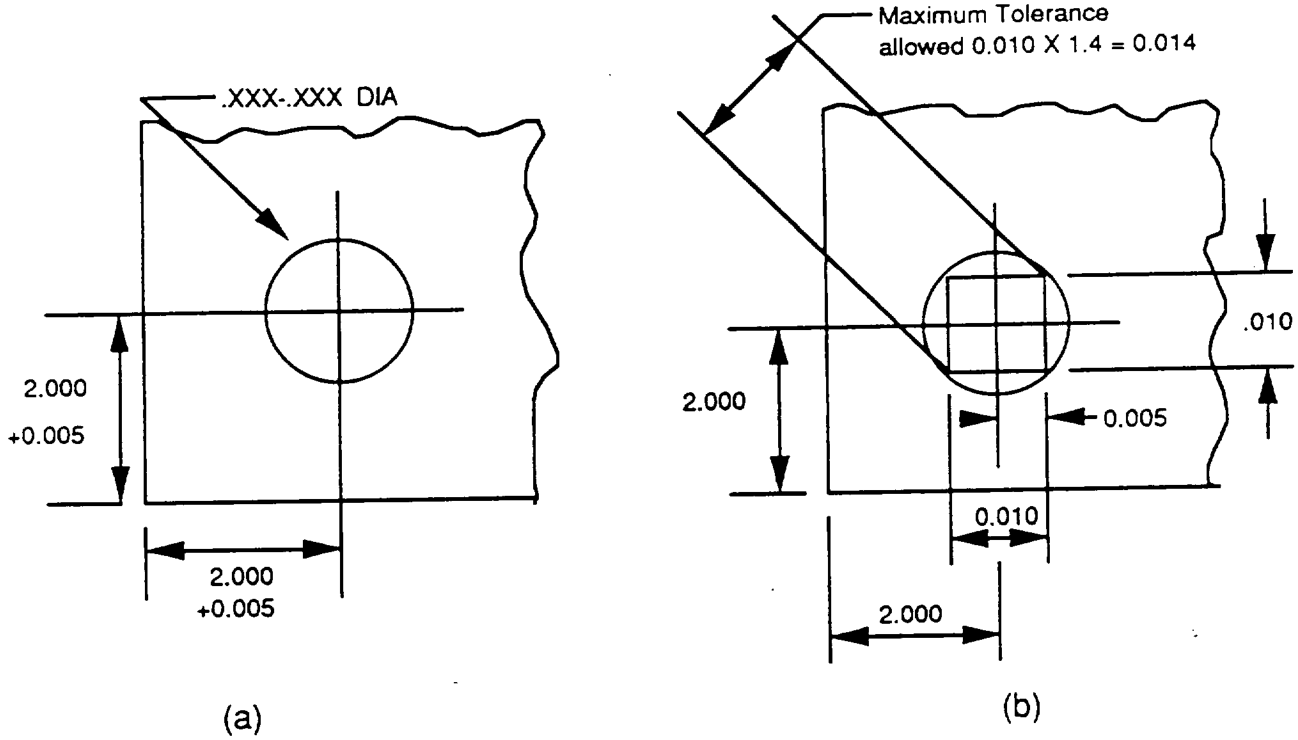


Figure 2.3. Tolerance zones (Giesecke et al, 1986).

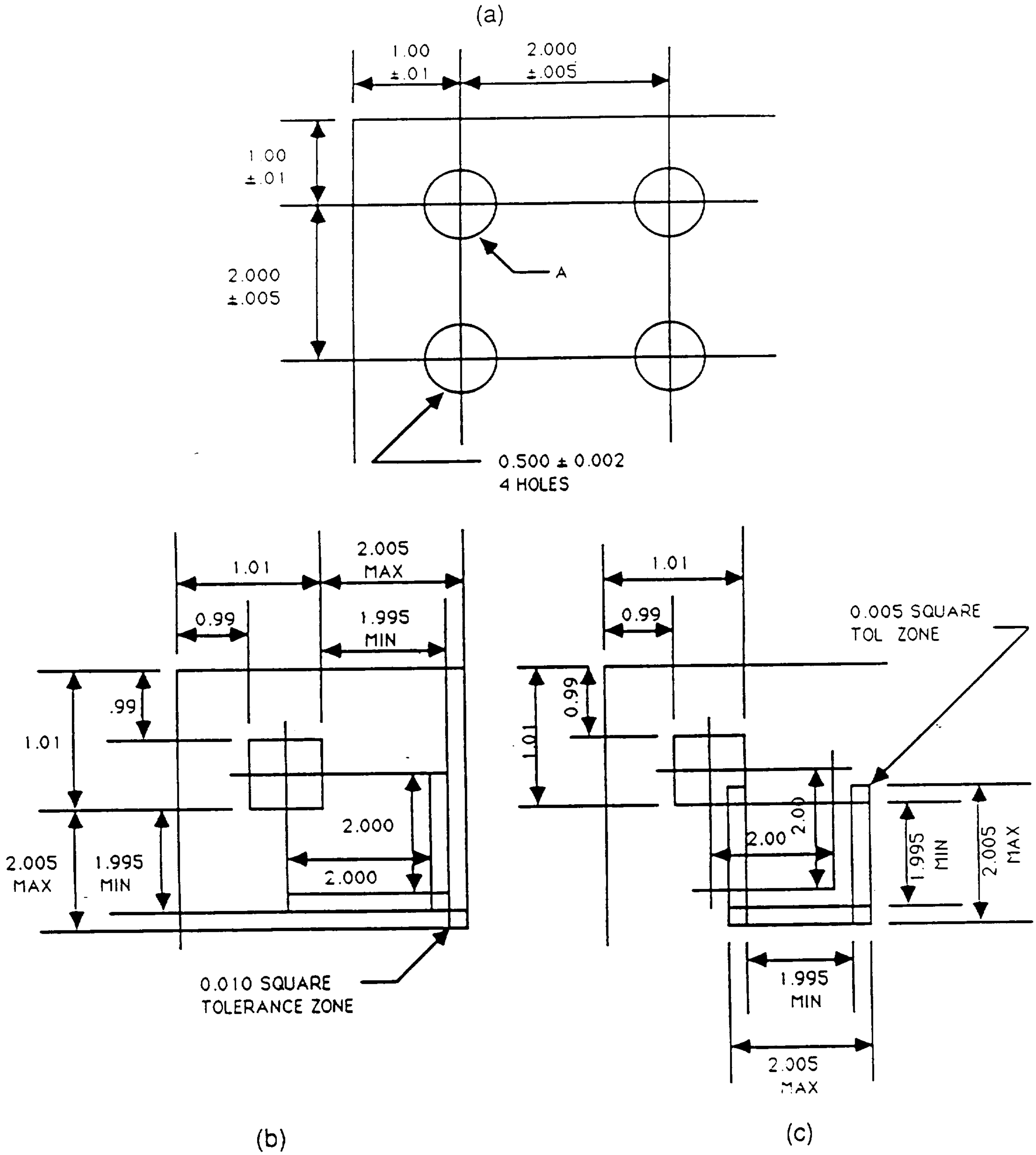


Figure 2.4. Tolerance zones (Giesecke et al, 1986).

the tolerances between the holes. The tolerances of these three holes will vary according to the actual position of the datum hole *A*. Therefore, it is difficult to say whether the resulting parts will actually fit the mating parts satisfactorily even though they conform to the tolerances.

2.1.4 Tolerance Zones

The above-mentioned disadvantage of getting an unsatisfactory fit even when parts conform to tolerances is overcome by giving exact theoretical locations by dimensions which are not toleranced and then specifying by a note how far actual positions may be displaced from these locations. This process is called true-position dimensioning. Using true positioning, the tolerance zone for each hole is a circle. The size of this circle will depend upon the variation permitted by the "true-position."

Actually, the "circular tolerance zone" is a cylindrical tolerance zone which is equal to the positional tolerance. This circular zone's axis must be within this cylinder as shown in Figure 2.5. The center line of the hole may coincide with or be parallel to the center line of the cylindrical tolerance as illustrated in Figures 2.5(a) and 2.5(b), respectively. The center of the line of the hole may also be inclined to the cylindrical tolerance while remaining within the tolerance cylinder as shown in Figure 2.5(c).

Requicha (1984) suggests this approach of using "tolerance zones." A part is said to be within specifications if its boundary is within these specified zones. As shown in Figure 2.6, three tolerance zones are necessary to specify the hole. The three zones describe size, position and form. A and B in the figure pertain to datum surfaces (normal drawing conventions). The algorithmic complexity will be increased for higher dimensional representations.

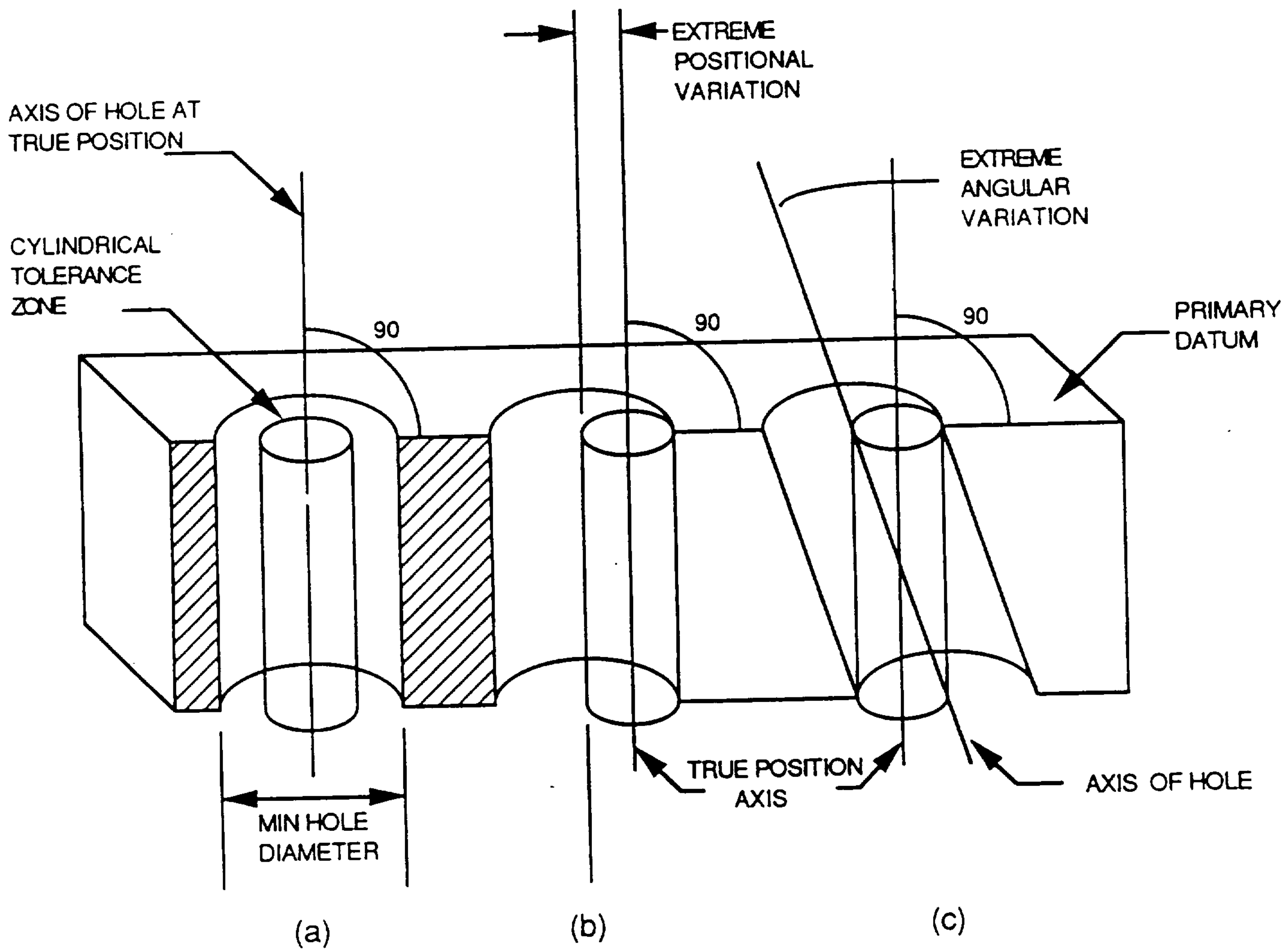


Figure 2.5. Cylindrical tolerance zones (Giesecke et al, 1986).

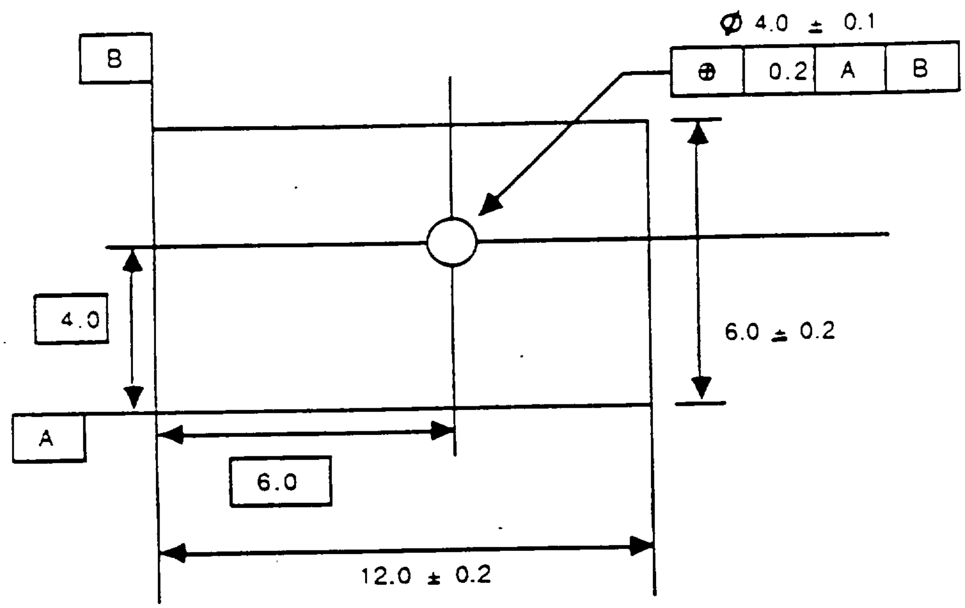


Figure 2. 6. Simple example of current tolerance practices (Requicha, 1993).

2.1.5 Implications on Manufacturability

The geometry of the manufactured part is always slightly different from the theoretically perfect design description models. Etesami (1988) describes a theory for constructing manufacturing part models. It is stated that theoretically perfect design description models do not always coincide with manufactured parts. Inspection tests determine conformance of the manufactured part with the design with respect to some acceptable deviations. Though design description models are efficiently performed by CAD systems, manufactured part modelling is non-existent. Thus, a unified theory is needed for computing the results of an inspection in building a geometric model. A manufactured part model can be used for tolerance verification, robot guidance in light assembly tasks, and for process control. Etesami presents a modeling scheme which proposes to integrate different forms of observations in order to construct a part model.

In theory, the proposed manufactured part model is composed of a boundary model, an axes (and curves) model, and a datum model. The part surfaces are represented by the bounding surfaces that envelop the measured part surface. Then, the surfaces have a perfect form similar to the nominal features. Thus, each surface has a boundary constructor enveloped by the MMC and the LMC surfaces (Figure 2.7). The curve and axis features are also represented by constructors as shown in Figure 2.8. The manufactured parts model must also include plane and axis equations that determine the position and orientation of datum planes and axes. Abstract descriptions and theories presented by Etesami to convey his concept are too detailed to be included in this report.

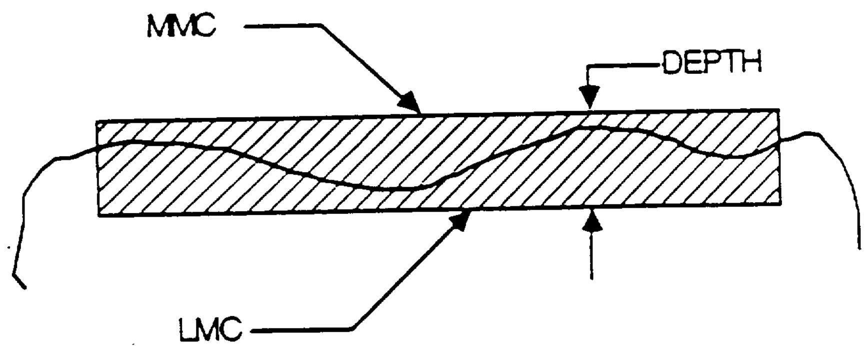


Figure 2.7. A planar surface constructor solid (Etesami, 1988).

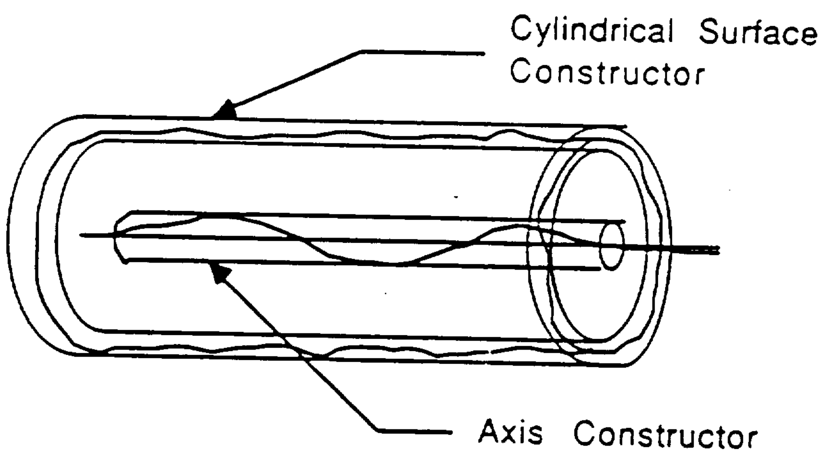


Figure 2.8. A cylindrical surface and its axis constructor (Etesami, 1988).

In summary, the imperfect features are modelled by constructor solids of perfect forms, the boundaries of which denote the extent of variations of the functional features. Tolerance specification statements are also explained in the light of the manufactured parts model.

2.1.6 Problems of Tolerancing: Solid Modeling

Here, attention is shifted to the solid modelers (described in section 2.1.5), which are widely used for Computer-aided Design (CAD). In the next few paragraphs, the tolerancing aspect for solid modelers is discussed in detail. According to Requicha et al (1982), a new generation of industrial geometry systems is emerging. The new systems are based on unambiguous solid models and thus are capable of supporting current and future applications automatically.

Requicha (1983) proposes a mathematical theory of tolerancing which formalizes and generalizes current practices and is a suitable basis for incorporating tolerances into the geometric (solid) modeling systems (GMSs). A tolerance specification in the proposed theory is a collection of geometric constraints on an object's surface feature. Three tolerancing constraints are considered viz., size tolerance, form tolerance, and position tolerance as shown in Figure 2.9. Figure 2.9(a) illustrates a size (radial) tolerance that varies from 4.95 to 5.05 about a basic dimension without specifying any other restrictions on form (surface). Figure 2.9(b), however, does not impose any size restrictions and specifies only a form tolerance. Figure 2.9(c) illustrates a tolerance zone with respect to a datum. The tolerance zone for any feature is obtained by "offsetting" the perfect-form surface. A feature can be located with respect to a master datum and used to construct a new datum. This new datum can be used to locate another feature.

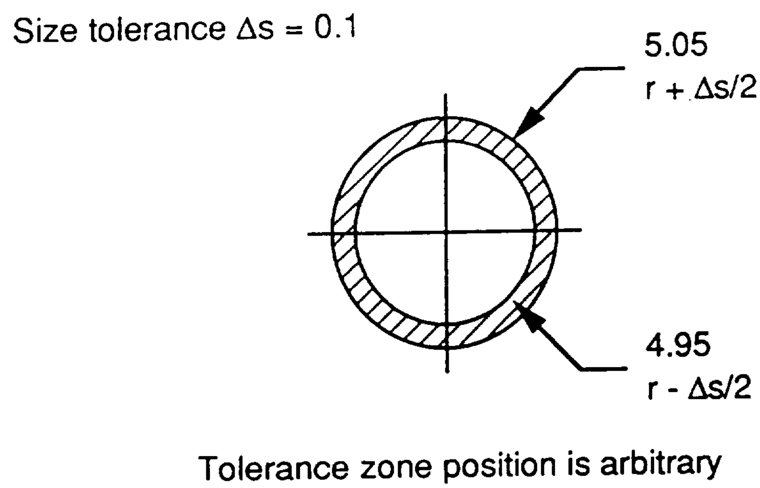


Figure 2.9(a). Tolerance zone for size (Requicha, 1983).

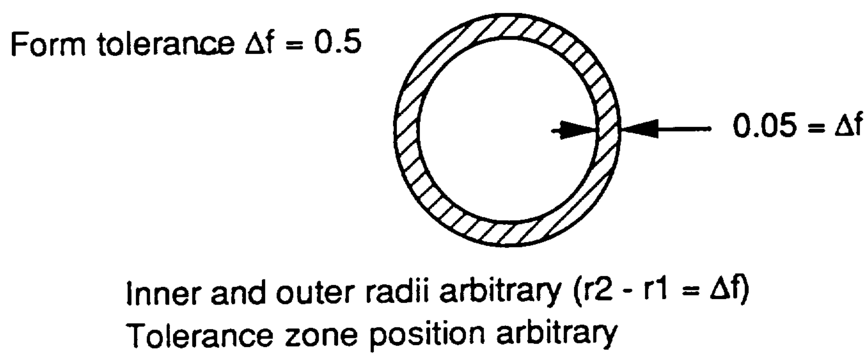


Figure 2.9(b). Tolerance zone for form (Requicha, 1983).

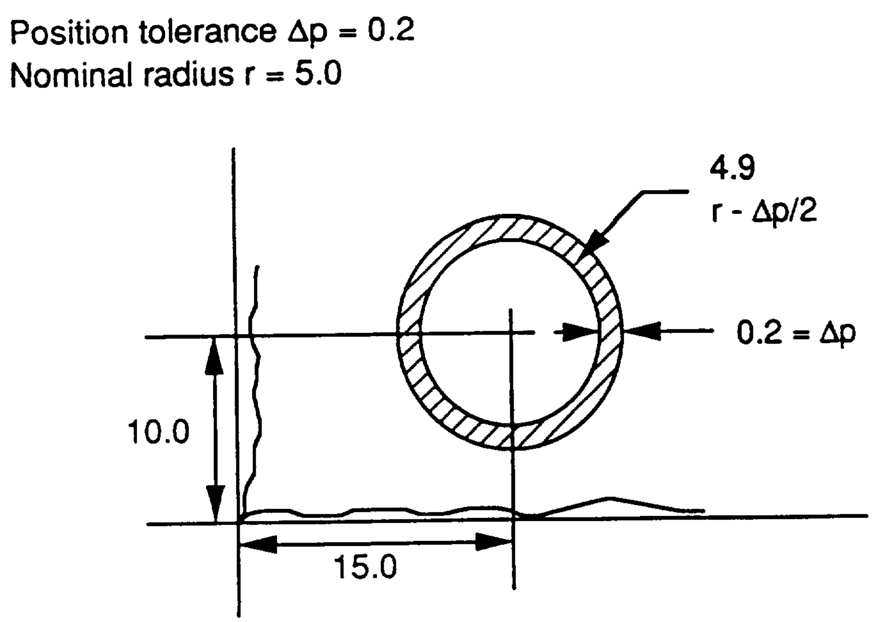


Figure 2.9(c). Tolerance zone for position (Requicha, 1983).

In the PADL-2 software developed by University of Rochester, Brown (1982) has a tolerance specification system which can provide the needed variational information for process planning. According to Brown, the premise of PADL-2 is that Geometric Solid Modeling Systems (GMSs) can be used for various purposes, and should be able to use internally some common representational and computational facilities. Internal representation pertains to the logical arrangement of data within the computer. The core system should cover 90 to 95 percent of typical unsculptured industrial parts. Brown's paper presents a modeling scheme which retains relevant manufacturing information of a product. The needed information can be captured through perfect form representation of surface features, curves and datum co-ordinate systems. For detailed analysis, refer to Brown (1982).

In an attempt to explain tolerances, Gossard et al (1988) present a method for representing dimensions, tolerances, and geometric features in solid models. The method uses a combination of Constructive Solid Geometry (CSG) and Boundary Representations (B-rep) in a graph structure called an Object Graph (Figure 2.10). The dimensions are represented by a Relative Position Operator (RPO). The RPO uses the dimension's nominal value to move an operand face with respect to a referent face (Figure 2.11). The positional tolerance for the dimension is stored using bounding limits. Gossard et al (1988) illustrated their approach for a polyhedral modeler. Their results were implemented in C on an IRIS 3030 workstation. Problems of stability and convergence (normally associated with numerical modeling) are avoided making their approach more robust than the variational geometry approach. Since our approach does not use this methodology, full details of the paper will not be discussed.

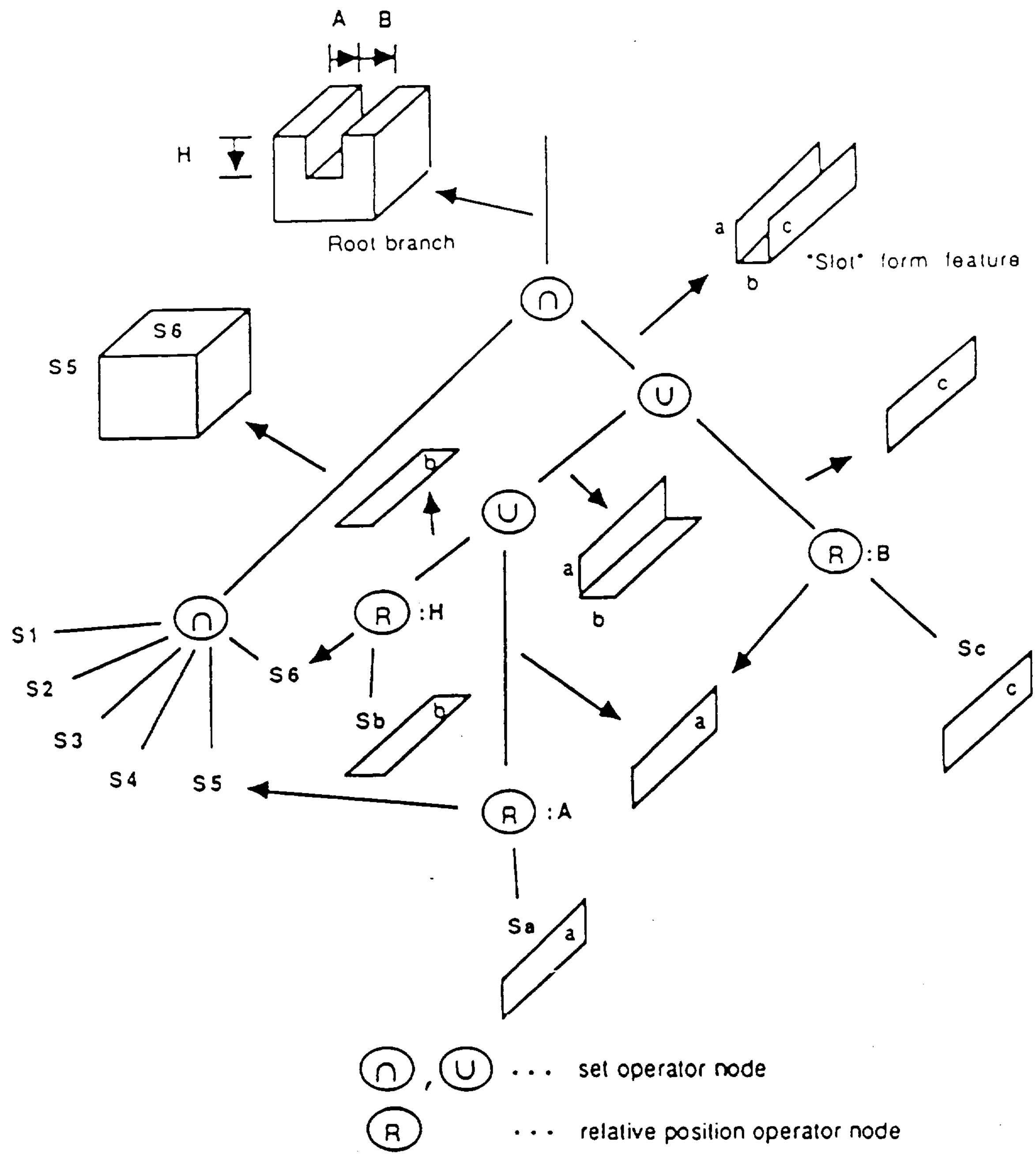


Figure 2.10. An object graph (Gossard et al, 1988).

Figure illustrates an object graph after Gossard et al.. The object graph has nodes and branches. The nodes represent operators and the branches denote B-reps of objects. The S's represent infinite half spaces. A,B and H denote dimensions and a,b,c arelabels of features.

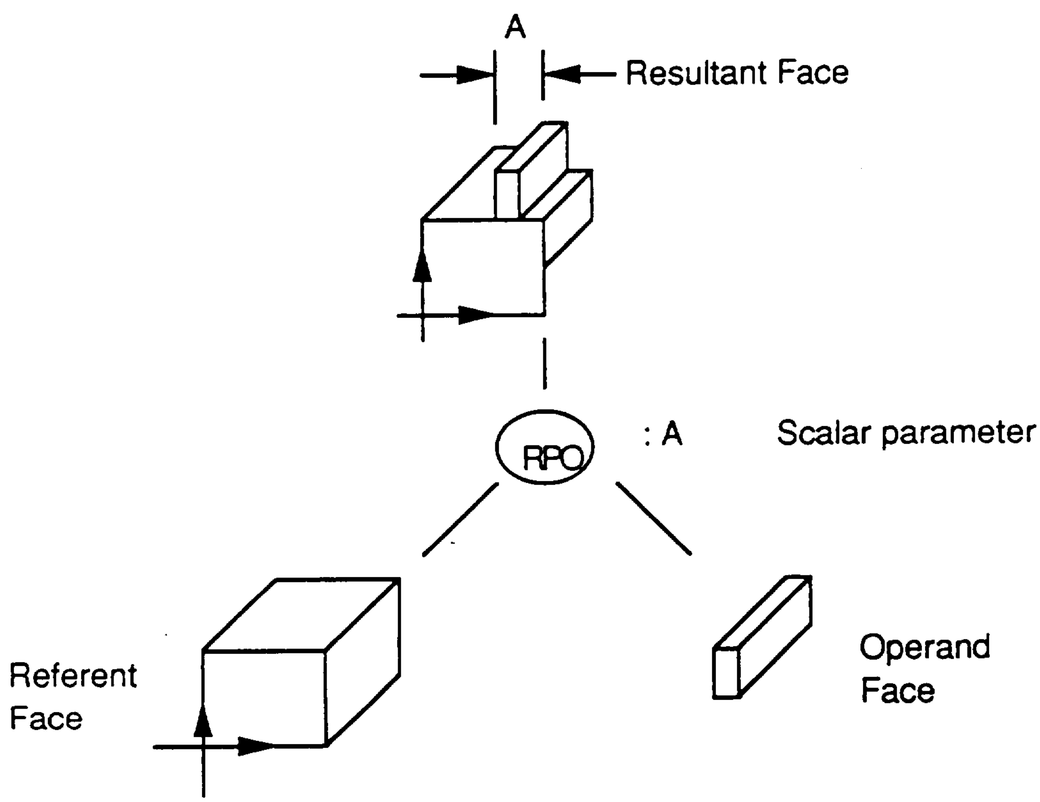


Figure 2.11. Relative position operator (Gossard et al, 1988).

Hillyard and Braid (1978) propose a theory to explain how dimensions and views combine to specify a shape. They also provide a method for determining whether a given dimensioning scheme is admissible. A method is suggested to discover what tolerances should be provided to the dimensions to achieve a desired precision. The results are developed for one, two and three dimensional spaces could be extended to spaces of higher order. This work would help in production of dimensioned views from stored shape descriptions. The shape model is obtained from a volume-based design system in which shapes are made by adding and subtracting such polyhedra as cubes and wedges. After the shapes have been made, the designer can provide dimensions and tolerances. Curved objects are not handled by the proposed scheme.

2.1.7 Conditional Tolerances

Srinivasan and Jayaraman (1985) explore the issues involved in a special case of tolerances called "Conditional Tolerances" for CAD systems. Conditional tolerances arise when the allowable deviations in the measured values of some geometrical parameters of a mechanical part depend on the measured values of some other geometrical parameters of the same part.

For fit and assembly or maintaining material thickness, a designer defines a virtual boundary. Then the requirements are set such that critical segments of boundary of the actual mechanical part must be "inside" or "outside" of the virtual boundary. In case of a fit and assembly, the virtual boundary requirement can be checked directly using functional gages. But, in maintaining material thickness, the virtual boundary requirement cannot be checked directly using functional gages.

A summary of some major issues in conditional tolerancing according to Srinivasan and Jayaraman (1985) is as follows:

(1) Representation of virtual boundary requirements.

(2) Building the converter to convert virtual boundary requirements to conditional tolerances. (This area needs further research.)

2.1.8 Tolerancing for Mating Parts

Tolerances pose more problems when the mating of two parts is to be considered. To carry out a specific function, the mating parts must satisfy the required type of fits.

Fits Between Mating Parts:

As mentioned before, fit is the general term used to signify the range of tightness or looseness which may result from the application of a specific combination of allowances and tolerances in mating parts (Giesecke et al, 1986). Giesecke et al (1986) define the four general types of fits between parts as:

(1) *Clearance Fit:* In this type of fit the internal member fits into an external member and always leaves an air space or clearance between the parts. Clearance fit is always positive.

(2) *Interference Fit:* In this type of fit the internal member is larger than the external member such that there is always an actual interference of metal. Interference fit is always negative.

(3) *Transition Fit:* This type of fit might result in either a clearance fit or interference fit.

(4) *Line Fit:* In this type of fit the limits of size are so specified that a clearance or surface contact may result when mating parts are assembled.

A system of different types of ANSI fits (ANSI B4.1-1967 (R1979)) is offered by Giesecke et al 1986. The solution of fits can be illustrated by the following example.

Using a hole basis system, suppose we wished to design a basic hole and shaft assembly (Figure 2.12a). When the design is begun, a general size (called the **nominal size**) of 2 inches is assigned to both the hole and the shaft diameters (Figure 2.12a) before considering the type of fit to specify. Later, when the design is further along, the 2 inch nominal size is replaced by a 2.000 inch **basic size** in order to make tolerance application possible. Then, when the designer is ready to factor in the fit specifications, both tolerances and allowances are applied simultaneously to the dimension in question. The allowances and tolerances make it possible to specify the acceptable variations in the dimensions and the desired "tightness" or "looseness" of the fit.

Suppose, now, that the designer wants a "medium" degree of clearance in the fit. A possible standard class of fit which could be used is the RC5 (for Running and Sliding Clearance) class of fit, which is approximately midrange between the tightest running/clearance fit (RC1) and the most open running/clearance fit (RC9). The RC5 tolerance specifications have been taken from the standard tables (American Association publication USAS B4.1-1967) and are summarized in the table below.

Table 2.1. RC5 standard fit specifications for 1.97-3.15 inch dimensions.

Nominal Size Range, **inches**		Clearance (Allowance)	Standard Tolerance Limits	
			Hole	Shaft
over	to	(values below in thousandths of an inch)		
1.97	3.15	+2.5 +5.5	+1.8 0	-2.5 -3.7

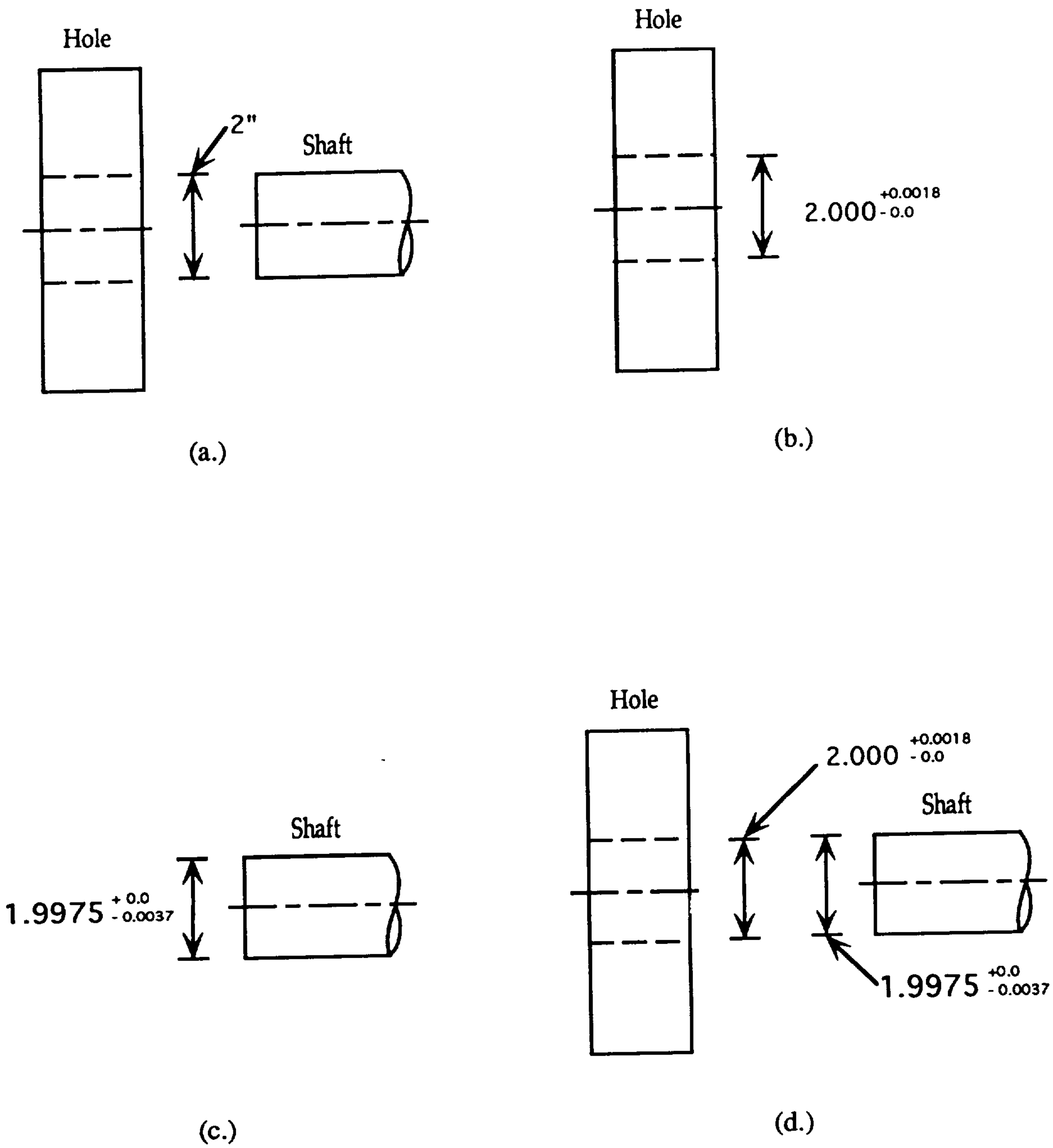


Figure 2.12. Example of the solution of fits for a hole and shaft assembly.

Since a hole basis system is being used, the hole is assigned the (standard) nominal size of 2.000 inches. With a clearance fit desired, the minimum clearance of .0025 inches is subtracted from the shaft's 2.000 inch nominal size to yield an adjusted shaft size of 1.9975 inches. The 2.000 inch hole size and the 1.9975 inch shaft size are now referred to as **design sizes** since they have been properly adjusted by the allowance and are ready to have tolerances applied to them. Finally, with the hole as the fixed basis for the assembly, the given hole tolerance specifications (Table 2.1) are directly applied to the 2.000 inch hole size (Figure 2.12b).

Since the shaft size has been adjusted by the minimum clearance (2.000" - .0025" = 1.9975"), the tolerances for the shaft must also be adjusted by the minimum clearance to yield the situation shown in Figure 2.12c. The final design (Figure 16d) takes into account both the dimensional accuracy and the intentional least difference (i.e., the "tightest" fit) permitted between two mating parts--which is referred to as the **allowance**.

According to Pollack (1988), other classes of fits which could have been used in the above example are:

- **Locational-Clearance (LC) fits**: intended for stationary assemblies which can be freely assembled or disassembled (always produces clearance and will not transmit motion)

- **Locational-Transition (LT) fits**: used where greater accuracy in assembly is needed (may produce clearance or interference and will not transmit motion)

- **Locational-Interference (LN) fits**: used where the accuracy of locating one part with respect to another is of primary importance (always interference, but not intended to transmit motion)

- **Force and Shrink (FN) fits**: produces a particular degree of force or shrink fit and may transmit motion

- **Running and Sliding (RC) fits**: (mentioned in the above example) intended to operate under running performance conditions when suitably lubricated

Computer Aided Fit Selection

Shifting from the main stream CAD tolerancing, the computer aided selection of fits and tolerances are described in the next few paragraphs. Papers by Lagodimos and Scarr (1983) and Lagodimos and Manalakos (1984) are briefly discussed. For detailed treatises, the reader is referred to the papers.

Lagodimos and Scarr (1983) describe the theory of interference fits. They also enumerate the parameters related to interference fits. Consequently, they outline a procedure for the computer-aided selection of interference fits using a microcomputer.

All materials undergo deformation under load. Up to a certain limit on loading, the deformation is temporary. This is to say, upon the removal of the load, the object regains its original shape. This type of deformation is elastic. However, with an increase in the load, the deformation becomes permanent (plastic deformation). An interference fit is a press fit which always necessitates deformation. Lagodimos and Scarr discuss the possibilities for deformation i.e., within elastic region or elastic-plastic region. It is suggested that two things have to be taken into consideration, when extreme limits of fit occur for a particular interference fit selected:

(1) The resulting maximum interference should not induce unacceptable stress conditions.

(2) The resulting minimum interference should be able to permit the transmission of the required load.

The computer-aided selection procedure proposed in this paper can be summarized by the following three statements:

(1) Calculations by a computer of the border interference limits i.e., maximum interference condition and minimum interference conditions that will allow selected fit to perform as per the requirements.

(2) Manual selection of standard tolerances and fundamental deviation from ISO/R286 tables which would satisfy required conditions.

(3) Calculations by computer of maximum and minimum load that the selected fit will be able to transmit.

Various inputs such as an inner diameter of a shaft, and an outer diameter of the hub are required for the program. Further, Lagodimos and Scarr describe in detail the method implemented in the computer program to calculate border limits of interference and transmissible load limits.

The program can be operated in two modes. In one mode the computer carries out the procedure for selection of interference fits. The other one determines the performance of the selected fit. The authors use the Newton-Raphson numerical iteration method as it offers a second order convergence thus ensuring rapid arrival at the solution.

In a subsequent paper, Lagodimos and Manalakos (1984) consider the problem of selecting ISO manufacturing limits to be assigned to parts that are to be connected through an interference fit. Figure 2.13 illustrates various aspects of an interference fit. The maximum and the minimum clearance (or interference) between the shaft and the hole are denoted by d_{max} and d_{min}, respectively. The tolerance zone for the hole is given by T_h. Similarly, T_s defines the tolerance zone for the shaft. The definitions for hole and shaft basis systems are discussed in detail later in the text (Section 2.1.10).

Various factors considered are:

(i) functional requirements of the end assembly,

(ii) manufacturing capabilities of the manufacturer,

(iii) limitations of the nature of ISO limits and fits.

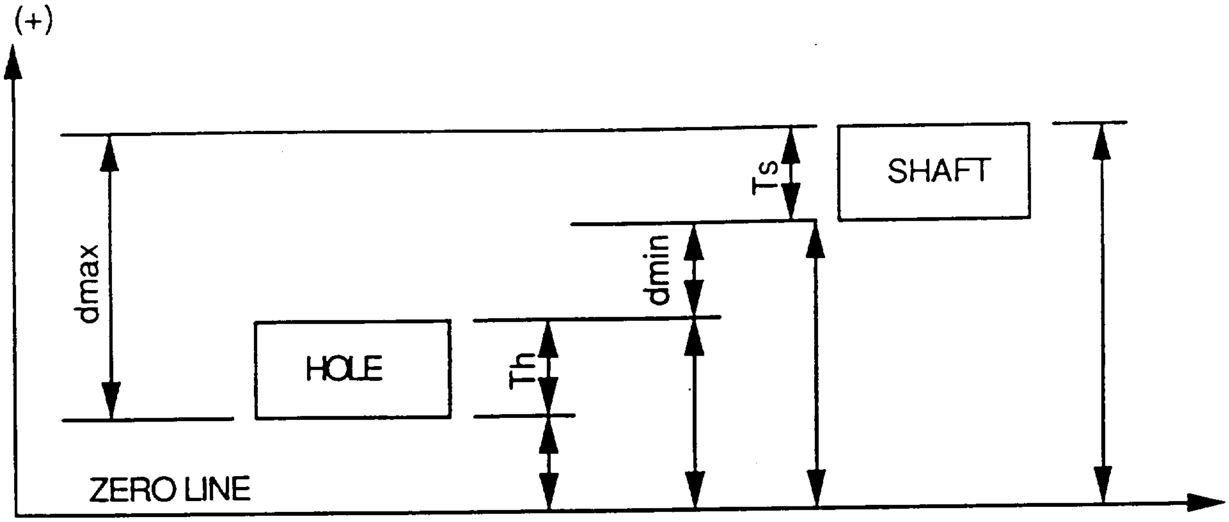


Figure 2.13(a). General representation of an interference fit (Manivannan et al, 1989).

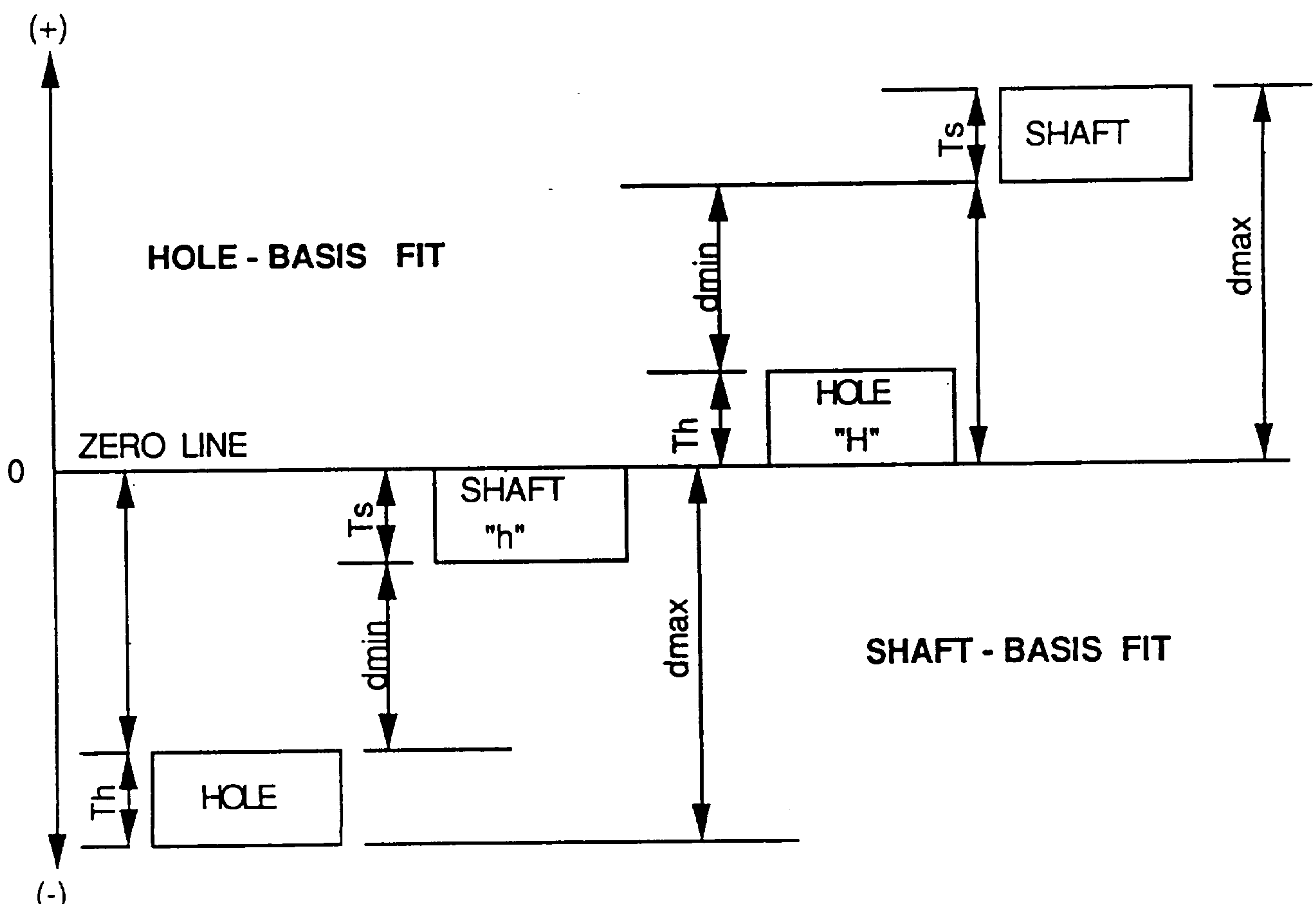


Figure 2.13(b). Hole-basis and shaft-basis representation of an interference fit (Manivannan et al, 1989).

After a calculation of the fit parameter values by the Cranfield approach, a rigorous method for computer aided selection of ISO Manufacturing Limits (MLs) is presented. This method would satisfy the existing functional and other requirements.

The user input consists of the following:

(1) Nominal diameter of the joint and the minimum and maximum border interference limits.

(2) Minimum tolerances (IT grades) for both the hole and the shaft that are producible. Also, the user has to provide preferences for the following:

(i) Larger tolerances allocated to any mating parts.

(ii) Selected fits given in hole-basis or shaft-basis system.

A determination is done to verify the existence of a satisfactory fit. If no fit exists, then the user is asked to slacken the initially specified load-carrying capacity. The program can tackle joint diameters of less than 500 mm.

2.1.9 Tolerance Allocation: Computer Programs

Tolerance programs presented by Ingham (1980), Patel (1980), and Dong and Soom (1986) are described in this section. These three papers have different objectives and cannot be compared on a one-to-one basis.

Ingham (1980) describes a computer program which analyzes the engineering tolerance. The common problems in computer aided tolerancing using an arbitrary feature chart of an object are pointed out in the paper. Note that features are geometric descriptors of specific regions of an object. In the feature chart given in Figure 2.14, F1, F2, F3 and F4 are the features that locate features R1 and R2. R1 and R2 are the features of interest at which clearances have to be found. The problems associated with tolerancing systems are:

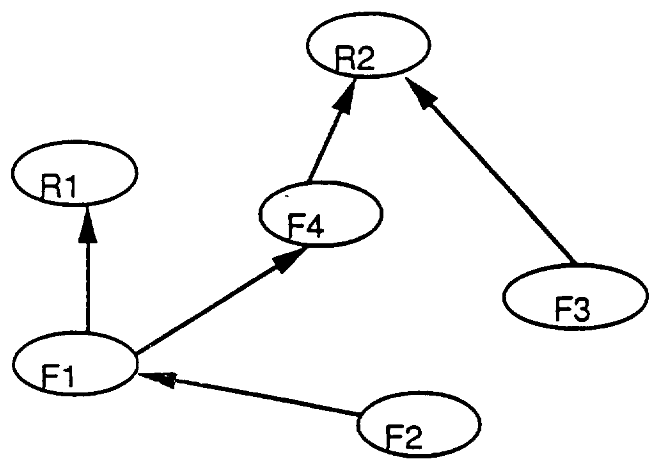


Figure 2.14. Engineering tolerancing for two dimensional case (Ingham, 1980).

(i) given unit tolerances acting at F1, F2, F3 and F4, the determination of the effects or sensitivity coefficients at R1 and R2;

(ii) given the tolerance distributions at F1, F2, F3, and F4, the determination of the cumulative distributions of tolerances at R1 and R2;

(iii) given constraints on the relationship between the actual tolerances acting at F1, F2, F3, and F4 and the tolerances of R1 and R2, the determination of the tolerances at F1, F2, F3, and F4 in a cost effective manner. These problems are implemented using a program ETA2.

Patel (1980) presents a quantitative model to consider tolerance capability of various processes used in the manufacture of a part and also the most effective way to combine tolerances in order to establish overall tolerances. The objective is to select the tolerances of the individual operations such that the sum of the cost is minimal, subject to the constraint that the resultant tolerance is within the design specification.

The cost-tolerance curves considered are non-linear. The tolerance for each operation is minimal. So, Patel (1980) linearizes the non-linear relationship by assuming a piecewise linearization of the whole curve.

The non-linear formulation of cost versus tolerance curves are approximated by an exponential function. Further, the author talks about tolerance assignment under Statistical Quality Control (SQC). SQC is applied when the cumulative effect of component tolerances on dimensions of the system is considered. Interactions of component tolerances on one another based on additive property of variances are considered under SQC, thus resulting in lower assembly costs.

Dong et al (1986) propose an Automatic Tolerance Analysis System. This system includes four phases. They are as follows:

(1) Automatic acquisition of all the related dimensions from a design database.

(2) Design assurance

(3) Tolerance distribution

(4) Design-to-manufacturing coordinate translation

After using this program one can apply more complicated statistical methods which decrease the manufacturing costs.

2.1.10 Tolerance Allocation: Using Knowledge Based System

This section discusses two papers that use knowledge based systems for manufacturing planning. While Ito et al (1988) paper discuss a framework for knowledge representation to generate part drawing and process plans, Manivannan et al paper (1989) emphasizes the task of specifying fits.

Ito et al (1988) discuss a methodology that will integrate the designer's knowledge and standards information (such as ISO) in order to prepare part drawings while simultaneously generating process plans. Discussions provided stress the need for setting up a knowledge and data base for storing standards and design/manufacturing information. The paper only discusses a simple prototype and is discussion oriented rather than implementation oriented. A simple example of the working of a trial system (prototype) is also included with relevance to a particular part drawing. This trial system also illustrates the storage of tolerance information specific to manufacturing.

Created to aid in the specification of ISO fits for the manufacture of rotational mating parts, the **ROSCAT** (**Rule Oriented System for Computer Aided Tolerancing**) system is discussed by Manivannan et al (1989). ROSCAT is capable of applying interference fits, clearance fits, and transition fits. Figure 2.15 gives examples of the three types of fits. The

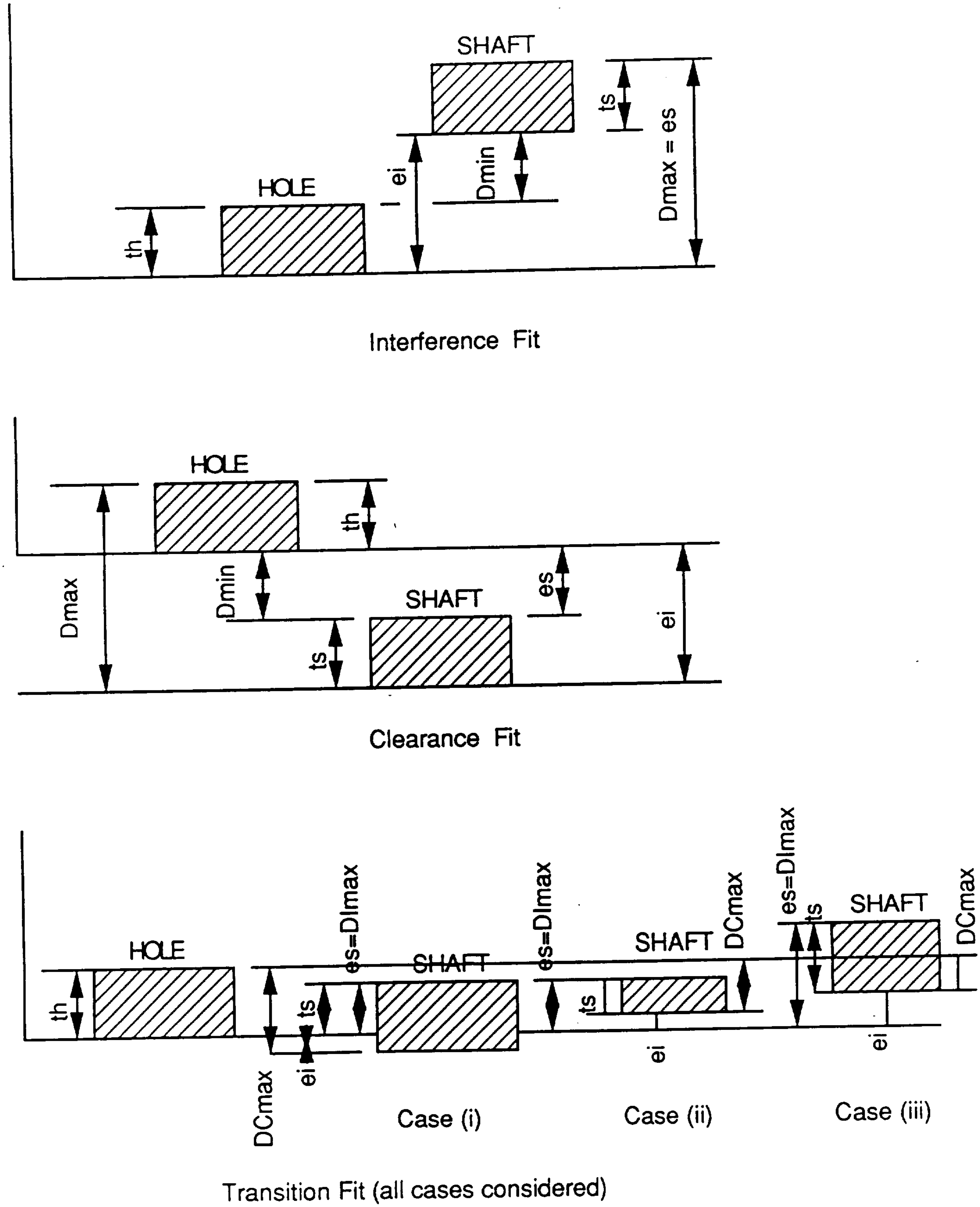


Figure 2.15. Various types of fits (Manivannan et al, 1989).

dimensions D_{max} and D_{min} represent the extreme allowances which are produced upon mating the hole and the shaft. Further, DI_{max} and DC_{max} represent maximum interference and maximum clearance in a transition fit. The hole and shaft tolerances (*th* and *ts*) as well as the fundamental upper and lower deviations (*es* and *ei*) are also shown. For the specification of tolerances, either the hole basis design system or the shaft basis design system could be used. Both of these systems are described below to help understand Manivannan et al (1989) contribution.

According to Pollack (1988), when two parts are designed to be assembled together, the desired degree of "tightness" or "looseness" of the assembly is referred to as the **fit** of the assembly. The standard way of looking at an assembly is to generalize the mating parts as being a "hole" and a "shaft"--thus one fits into the other. When making this generalization and applying fits, the designer most often uses a **hole basis system**, i.e., the hole size is taken as a given and the shaft size is adjusted according to the desired fit. Conversely, a **shaft basis system** could also be used--where the shaft size is fixed and the hole is adjusted. However, the hole basis system is the most common design technique since bores, drills, etc. come mainly in standardized sizes--making it much easier to let the shaft take on the non-standard size.

Basic Hole System:

As shown in Figure 2.16(a), the minimum size of a hole 0.600 inch is taken as basic size. A specified allowance of 0.002 inch is subtracted from 0.600 inch which gives a maximum shaft of 0.598 inch. Tolerances of 0.002 inch and 0.003 inch respectively are applied to the hole and shaft to obtain maximum hole of 0.602 inch and minimum shaft of 0.595 inch. So, the minimum clearance between the parts becomes 0.600 - 0.598 = 0.002 inch and maximum clearance is 0.602 - 0.595 = 0.007 inch.

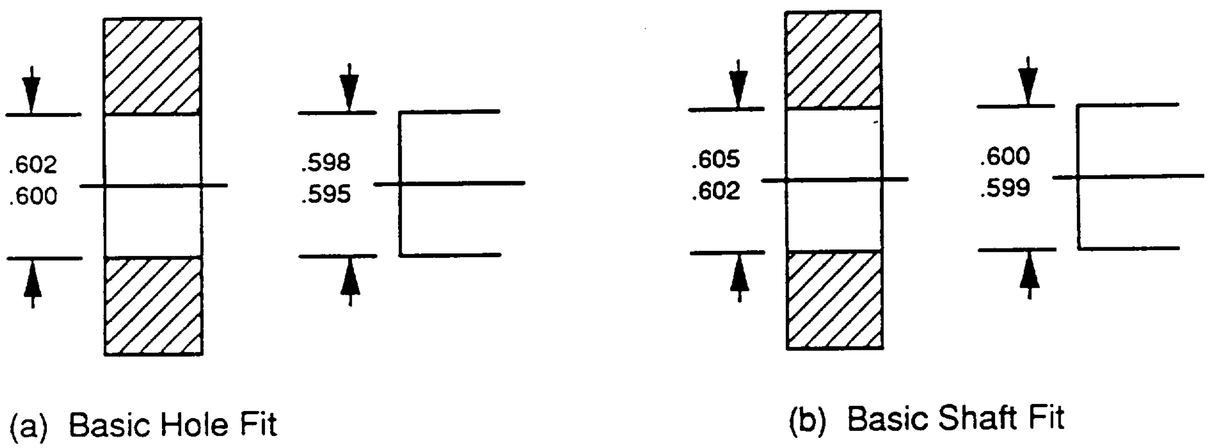


Figure 2.16. Basic hole and basic shaft systems (Giesecke et al, 1980).

Basic Shaft System:

As shown in Figure 2.16(b), the maximum size shaft is 0.600 inch. This maximum size is taken as the basic size. A specified allowance of 0.002 inch gives a minimum hole of 0.602 inch. Tolerances of 0.003 inch and 0.001 inch, respectively, are applied to the hole and the shaft. This gives maximum hole of 0.605 (0.600+0.002+0.003) inch and minimum shaft of 0.599 (0.600-0.001) inch. Therefore, the minimum clearance between parts is 0.002 (0.602-0.600) inch and maximum clearance is 0.006 (0.605-0.599) inch.

ROSCAT is a rule based system for tolerance allocation. Standard data on fits and tolerances are stored in a data base. The rules that link the shaft/hole diameter and other dimensions to the tolerance on the hole/shaft have been formulated and input into the system. Given a particular shaft/hole diameter, the system allocates tolerances for best performance of the fit according to the ISO specifications.

2.1.11 Problems in Representation of Tolerances: Miscellaneous

Two miscellaneous articles which could not be included in the preceding sections are discussed here.

Truslove (1988) points out that one of the major benefits in integrating CAD and CAM is the potential improvement in information flow between design and manufacturing activities. This information flow should be such that information on production capabilities passes back to the design stage.

Tolerances have no direct significance on the component geometry. So, these tolerances are generally regarded as an annotation on the manufacturing drawings. Further, when geometric data is transferred from CAD to CAM, the textual information is omitted making some tolerance information inaccessible for manufacturing (Figure 2.17). Hence, tolerances should be stored as geometric attributes. Truslove emphasizes the need for retrieving tolerance information from geometric definitions and states that data integrity is dependent on the same. Some other issues of importance, according to Truslove, are:

(1) Accuracy of information:

> After a series of edits or transformations on geometric data, computational rounding errors would accumulate. These errors eventually can alter a geometric size so much that they may exceed manufacturing tolerance limits as shown in Figure 2.18. This depends on the precision of the arithmetic used by the CAD software.

(2) Effect of databases on the re-use of data:

> At present, there are three possible sources of information available which can describe the component. These are a 'paper' or 'hard-copy' database, a CAD based electronic database and a CAM based electronic database. The hardcopy data base essentially consists of production drawings and the CAD and CAM based data bases contain digital representations of geometry. Now, the question arises as to which definition should be used as the master source of data.

(3) Re-use of geometric definitions:

> There are two areas of re-use within the design and manufacturing environment. These two areas are: during development of a complete product design and in the automated methods for production, planning and inspection.

Truslove states that a tolerance actually represents spatial variation between two or more geometric elements rather than the variation in size on a single geometric element as shown in Figure 2.19. This brings up the problem of how to store the tolerance information. The problem of tolerance attribution in the context of a two dimensional drawing has not been fully explored yet. Therefore, Truslove states that tolerance attribution should be indirectly applied through the principle of dimensioning in two dimensional drawings as shown in Figure 2.2.

A component cannot be defined by a single orthographic view, but it is difficult to link an increased number of views in a CAD system. So, tolerance attribution becomes more complex.

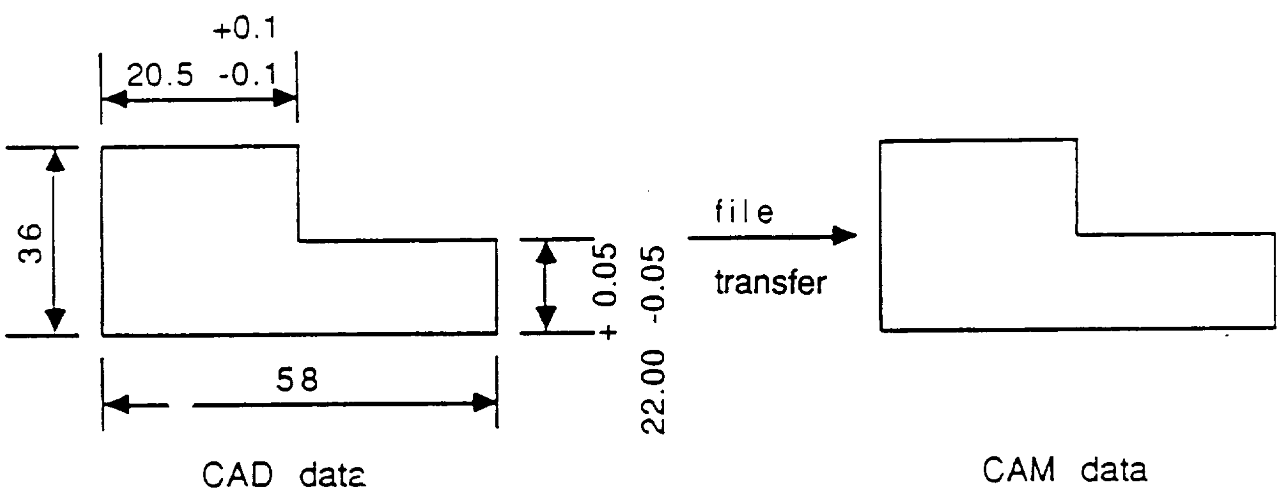


Figure 2.17. Removal of data during file transfer (Truslove, 1988).

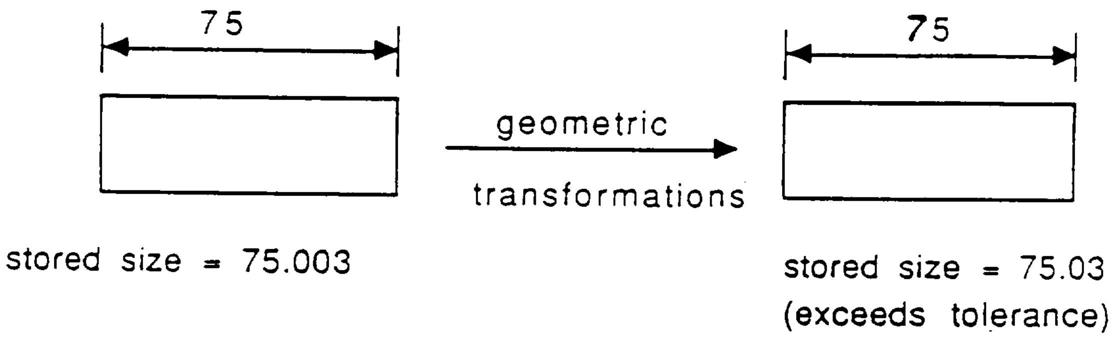


Figure 2.18. Effect of repeated geometric transformations (Truslove, 1988).

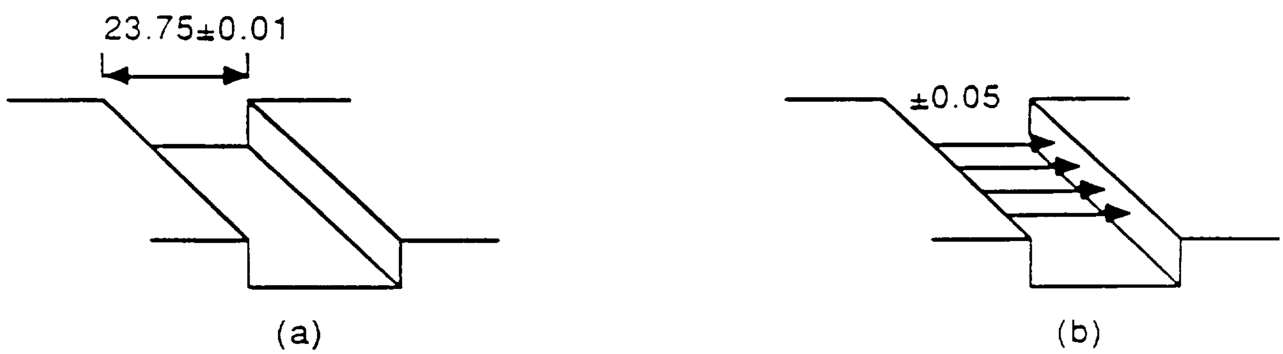


Figure 2.19. Attribution of tolerances for: (a) tolerance as a dimensional attribute, and (b) tolerance as a geometrical attribute (Truslove, 1988).

To sum up, Truslove emphasizes the need for data re-use throughout the design and manufacturing processes so as to avoid multiple definitions. He also stresses the implications of tolerance representation within CAD systems.

The lack of facilities for representing tolerances and related information is a major deficiency of contemporary solid modelers. Requicha (1984) discusses the semantics of tolerancing for mechanical parts. Particular emphasis has been made on the problems arising when features of physical objects cannot be assumed to have perfect form.

Incorporating tolerancing information in solid modelers raises the following three main issues. They are as follows:

(1) Representation of tolerances:

How can tolerances be represented in modelers?

(2) Analysis and synthesis of tolerance specifications:

For a given two toleranced parts, are there instances that parts "in spec" will interfere with each other and therefore fail to assemble?

(3) Applications of tolerancing information:

How are tolerancing data to be used for automatically planning the manufacture, inspection and assembly of mechanical components?

A variational class is a family of objects. The perfect form semantics imply that only perfectly shaped objects are within specifications. This implication clearly violates the condition that a tolerance specification should not force any portion of an object's boundary to be perfect. Therefore, perfect-form semantics do not provide a suitable basis for representing tolerances in solid modelers.

The imperfect-form approach could be dealt with by using a surface-fitting technique in conjunction with perfect-form semantics. This approach could be useful for establishing datum, but unsuitable for other requirements of modern tolerancing.

An alternative approach is to use "tolerance zones" and say that a part is in spec if its boundary lies within such zones. The tolerance zones are named for size, position and form. But, then a problem arises when a zone has to be specified for a three-dimensional feature.

Two approaches, a parametric and a non-parametric approach are specified by Requicha (1984). In the parametric approach, parameterization must be specified by the user. It is difficult to separate the definition of a nominal shape from the specification of tolerances. The reader is referred to Requicha (1984) for in-depth details.

The non-parametric approach is simpler since it bypasses all problems associated with parameterization. It permits a clean separation between nominal shape definition and tolerancing. This approach is easier to implement and is independent of how features are represented.

Tolerance information is essential for planning part manufacturing and assembly operations. However, automatic tolerance analysis is not yet available on Computer Aided Design and Manufacturing systems. Dimensions are usually not independent. It is therefore important to study the relationships among dimensions, their tolerances and their effects on the final manufactured or assembled product.

2.2 Tolerance Optimization

The research on tolerance specification is divided into two groups:

(1) Tolerance Analysis

(2) Tolerance Allocation

In tolerance analysis the component tolerances are all known or specified and the resultant tolerance is calculated. In tolerance allocation, however, the assembly tolerance is known and the component tolerances are allocated.

2.2.1 Tolerance Analysis

Component tolerance requirements may be determined using ANSI standards, company manuals, and government regulations. The calculation of an assembly tolerance from component tolerances is not a trivial task. It is desired to predict assembly tolerance close to the actual assembly tolerance limits so that sound predictions can be made on the number of rejects or scrap. A study by Wu et al (1988) discuss eight tolerance analysis models cited in the literature. Brief review and comparisons of these techniques is discussed in this tolerance analysis section. The eight tolerance analysis techniques are:

1) Worst Case Model

Let t_i denote the specified tolerance for component i, i=1,..., n. Then, t_a, the assembly tolerance is calculated as

$$t_a = \sum_{i=1}^{n} t_i$$

This method produces the most conservative assembly tolerance value. The calculated assembly tolerance is much higher than the actual assembly tolerance. Inflated assembly tolerance forces individual allocated component tolerances to be very tight in order to reduce the assembly tolerance. This results in high manufacturing costs. This method is favored, however, when all assemblies must be within allowable limits and no rejects are permissible.

2) Statistical Model

This model is also referred to as the Root Sum Squares (RSS) model. It can, in the general sense, be represented by

$$t_a = z_a \left[\sum_{i=1}^{n} \left(\frac{t_i}{z_i} \right)^2 \right]^{0.5}$$

where z_a = assembly deviation multiplier, and

z_i = deviation multiplier for the *i'th* component tolerance.

For the normal distribution, the deviation multiplier is set to 6 when 6σ limits are considered. This model produces smaller assembly tolerance, especially, when the number of components is large, or when the individual component tolerance distributions are symmetric around the tolerance midpoint.

3) Spotts' Modified Model

This model is a combination of the worst case model and the statistical model with z_a and z_i set to 6 for all *i*. The assembly tolerance, t_a, is calculated as:

$$t_a = 0.5 \left(\sum_{i=1}^{n} t_i + \left(\sum_{i=1}^{n} t_i^2 \right)^{0.5} \right)$$

The weight of the previous two models in the above formula are equal as indicated by 0.5 in front of the outermost parenthesis. For skewed distributions this model determines an assembly tolerance which leads to fewer rejects.

4) Modified Statistical Model

This model takes into account nonrandom factors such as errors in predicting component tolerance distributions. A correction factor, c_f, is introduced which is to cover up sum of the process errors. Several correction factors as a function of the number of components and the number of rejects have been suggested by Wolff (1961). However, Wu et al (1988) suggest a correction factor value of 1.4 or 1.5. The modified statistical model is given by

$$t_a = c_f\, z_a \left(\sum_{i=1}^{n} \left(\frac{t_i}{z_i}\right)^2\right)^{0.5}$$

As Greenwood and Chase (1987) have pointed out, a major shortcoming of this model is its lack of physical significance. When the component tolerance distributions are symmetric and the number of components is large, the method generates assembly tolerances that are both feasible and desirable. However, the assembly tolerances do not necessarily enhance quality thus making the method a poor tool for quality improvement.

5) Mean Shift Model

Mean shifts in tolerance distributions can arise due to tool or die wear and variability in production processes. The model suggested by Greenwood and Chase (1987) incorporates a mean shift (bias from the tolerance midpoint value) and the variability around the mean into the formula. The model is described by

$$t_a = \sum_{i=1}^{n} m_i t_i + \left(\sum_{i=1}^{n} [(1-m_i)t_i]^2\right)^{0.5}$$

where m_i is the possible range of mean shift for the i'th component (expressed as a function of its range).

The value of m_i is difficult to predict, especially, when data about component tolerance distributions are not available. This model is preferred when few or poorly controlled components (high process variability) define the assembly.

6) Monte-Carlo Model

This model assumes that component tolerance distributions are known. A simulation program randomly and repeatedly generates component dimensions using component tolerance distributions and calculates assembly dimension. Let X_{max} and X_{min} denote the largest and smallest assembly dimensions encountered by the simulation program, respectively. Then, the assembly tolerance is determined using

$$t_a = X_{max} - X_{min}$$

Wu et al (1988) point out that when the number of comparisons is large, the assembly tolerance reduces significantly due to the low probability of generating extreme dimensions for all components at the same time by the Monte-Carlo simulation program. This model is a good tool when component tolerance distributions are known but too complicated for mathematical analysis. The disadvantage of the model is the requirement of excessive computer times.

7) Moment Model

In this model, the component maximum dimension (X_{max}) and the minimum dimension (X_{min}) are calculated using mean and standard deviation of component tolerance values. More specifically,

$$X_{max} = m_a + 3\sigma_a$$

$$X_{min} = m_a - 3\sigma_a$$

where $m_a = \sum_{i=1}^{n} m_i$

$$\sigma_a = (\sum_{i=1}^{n} \sigma_i^2)^{0.5}$$

and

$$t_a = X_{max} - X_{min}$$

m_a = assembly mean tolerance

σ_a = standard deviation of the assembly tolerance

m_i = component mean tolerance

σ_i = standard deviation of the *i'th* component tolerance.

This model is preferred over the Monte-Carlo model when components are identically distributed and the mean and standard deviations of each component tolerance distribution can be calculated.

8) Hybrid Model

This model combines the Monte-Carlo model and the moment model. It uses Monte-Carlo simulation to generate one thousand assembly tolerances given by x_i, i=1,...,1000 which are used to calculate m_a and σ_a used in the moment model. More specifically,

$$m_a = 0.0001 \sum_{i=1}^{1000} x_i$$

$$\sigma_a = (0.0001 \sum (x_i - m_a)^2)^{0.5}$$

$$X_{max} = m_a + 3\sigma_a$$

$$X_{min} = m_a - 3\sigma_a$$

The model requires less computation time and is shown by Wu et al (1988) to predict relatively larger rejects than the Monte-Carlo model, but still smaller than most of the other models such as Worst-Case model, Spotts' statistical model and moment model.

2.2.2 Tolerance Allocation

The tolerance allocation problem is the problem of allocating component tolerances while observing the total assembly tolerance in a way to minimize total cost. Generally, mathematical models are developed for the problem and solved using optimization techniques. Mathematical models are mathematical representations of the problem indicating the overall objective (which is generally the minimization of the total cost) and constraints of the problem. Review of literature indicates that workable mathematical models have been developed for simplified problems and these models have been solved using existing optimization techniques. Speckhart (1972), Ostwald and Huang (1977), Patel (1980), and Chase and Greenwood (1988) discuss tolerance allocation models that focus on the minimization of the total cost subject to either a constraint on the mean assembly tolerance requirement or a constraint on the variance of the assembly tolerance distribution. Wu et al (1988) also discuss some of these models. Peters (1970), Michael and Siddall (1981, 1982), and Parkinson (1982, 1984, 1985) demonstrate mathematically more complicated tolerance allocation models through examples. However, it is not easy to describe these models for a generic tolerance allocation problem. Although a few comments will be made on these models, the details and mathematical expressions concerning the models will be omitted from this report.

One approach to tolerancing based on minimum cost is presented by Speckhart (1972). This paper presents a workable analytical method for finding the optimum set of dimension tolerances for a mechanical device that will minimize manufacturing costs and meet the imposed constraint conditions.

Using the mathematical description of the constraints and information on the cost of manufacturing of each dimension as a function of the tolerance, the method utilizes Lagrange multipliers to minimize nonlinear cost functions subject to linear/nonlinear constraints. In order to use this method, the following three issues have to be addressed:

(1) Mathematical description of the critical dimensions

In a shaft and hole assembly with a shaft of dimension t_1 and a hole of dimension t_2, the diametrical clearance is given by

$$t_a = t_1 - t_2$$

The above equation is referred to as tolerance assembly function.

(2) Tolerances of the critical dimensions

For our shaft and hole assembly, the maximum allowable t_a has been determined beforehand. For example,

$$t_a \leq 0.005$$

(3) Cost and tolerance curves

Speckhart assumes an exponential relationship between the cost of manufacturing and component tolerances. The relationship can be mathematically expressed as follows:

$$c_a = \sum_{i=1}^{n} (A(i) + B(i)e^{C(i)t_i})$$

where c_a = total cost of the assembly

$A(i)$, $B(i)$, $C(i)$ = Constants for the *i'th* component

t_i = tolerance of the *i'th* component.

In general, the assembly tolerance function can be written as a function of component tolerances as

$$t_a = f\,(t_1, t_2, \ldots, t_n)$$

A constraint that will insure the total assembly tolerance to be within the desired range is needed in the model. This type of constraint can be expressed as

$$\sum_{i=1}^{n} \left|\frac{\partial t_a}{\partial t_i}\right| t_i \leq t_{as}$$

where $t_{as} \equiv$ maximum allowable assembly tolerance. For the hole and shaft assembly,

$$\frac{\partial t_a}{\partial t_1} = 1, \qquad \frac{\partial t_a}{\partial t_2} = -1.$$

Therefore, the constraint is

$$\left|\frac{\partial t_a}{\partial t_1}\right| t_1 + \left|\frac{\partial t_a}{\partial t_2}\right| t_2 \leq t_{as}$$

That is,

$$t_1 + t_2 \leq 0.005$$

The overall nonlinear tolerance allocation model proposed by Speckhart can be summarized as:

$$\text{Minimize} \quad c_a = \sum_{i=1}^{n} \left(A(i) + B(i)e^{C(i)t_i}\right)$$

$$\text{Subject to} \quad \sum_{i=1}^{n} \left|\frac{\partial t_a}{\partial t_i}\right| t_i \leq t_{as}$$

The previous model is called a nonlinear programming model due to the existence of nonlinear functions in the model. There exists several optimization techniques which can solve this model. One of these techniques is called the Lagrangean technique which is used by Speckhart to arrive at a closed-form solution for the model. Speckhart's model is a good tool when exponential cost function is appropriate. The model doesn't take the process variability into account.

Speckhart (1972) also proposed another type of constraint of the form

$$\sum_{i=1}^{n} \left|\frac{\partial t_a}{\partial t_i}\right| t_i^2 \leq t_{as}^2$$

which insures the tolerances to be within the 3σ limits of the assembly tolerance. However, since the constraint is nonlinear, solution techniques cannot guarantee absolute minimum for the objective function.

Chase and Greenwood (1988) consider a similar tolerance allocation model. They provide closed form solutions to their model where the tolerance-cost function is assumed to be a function of the reciprocal of the component tolerances rather than exponential. The mathematical model is expressed as:

$$\text{Minimize} \quad c_a = \sum_{i=1}^{n} \left(A(i) + \frac{B(i)}{t_i}\right)$$

$$\text{Subject to} \quad \sum_{i=1}^{n} t_i^2 \leq t_{as}^2$$

This model is appropriate for tolerance allocation problems with the cost function given by the expression for c_a.

A rather more simplified linear model is discussed by Patel(1980). Patel's model assumes linear cost-tolerance function with the slope being $B(i)$ for the $i'th$ component. It also assumes upper and lower limits on component tolerances which are dictated by the manufacturing processes. The mathematical model is given by

$$\text{Minimize} \quad c_a = \sum_{i=1}^{n} (A(i) - B(i)t_i)$$

$$\text{Subject to} \quad \sum_{i=1}^{n} t_i \le t_{as}$$

$$t_{li} \le t_i \le t_{ui}$$

where t_{li} = minimum allowable tolerance for component i,

t_{ui} = maximum allowable tolerance for component i.

As an example, consider a three component assembly. The model can be written as:

$$\text{Minimize} \quad c_a = (1\text{-}80t_1) + (2\text{-}25t_2) + (5\text{-}50t_3)$$

$$\text{Subject to} \quad t_1 + t_2 + t_3 \le 0.005$$

$$0.001 \le t_1 \le 0.002$$

$$0.0005 \le t_2 \le 0.003$$

$$0.0015 \le t_3 \le 0.003$$

The above model can be optimized using the linear programming technique and the optimal solution is found to be:

$$t_1 = 0.002,\ t_2 = 0.0005,\ t_3 = 0.0025, \text{ with } c_a = 7.703.$$

Patel also investigates nonlinear tolerance optimization models (the same models proposed by Speckhart) and uses the Newton-Raphson optimization technique to determine the best component tolerance values. Techniques such as the Newton-Raphson technique yield good solutions for small size problems. For moderate size problems heuristics may be a better fit.

Ostwald and Huang (1977) study the tolerance allocation problem in a way which is different from the models discussed by Patel (1980), Speckhart (1972) and Chase and Greenwood (1988). Ostwald and Huang assume a simple assembly model where component dimensions are added to determine the assembly dimension. Each component can be produced using different manufacturing processes. They propose a linear model whose solution will indicate the optimal process selection for each component such that t+w assembly tolerance will not be exceeded and the manufacturing cost is minimized. The model can be mathematically expressed as:

$$\text{Minimize} \quad c_a = \sum_{i=1}^{n} \sum_{j=1}^{t(i)} c_{ij}\, x_{ij}$$

$$\text{Subject to} \quad \sum_{i=1}^{n} \sum_{j=1}^{t(i)} t_{ij} x_{ij} \le t_{as}$$

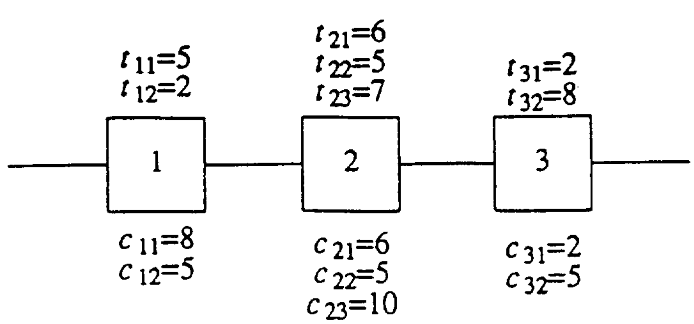
where c_{ij} = cost of manufacturing process j to produce component i

t_{ij} = tolerance for *i'th* component and the *j'th* manufacturing process

t_{as} = overall assembly tolerance

x_{ij} = decision variable which selects the *j'th* process for component i.

To illustrate the above model, consider three components assembled in series, as shown below:



Components 1 and 3 can be processed in two different ways. The component tolerances (t_{ij}) and related cost (c_{ij}) for each component and each process is shown in the figure. Let the maximum assembly tolerance be 20. The corresponding mathematical model can be written as:

$$\text{Minimize} \qquad c_a = 8x_{11} + 5x_{12} + 6x_{21} + 8x_{22} + 10x_{23} + 2x_{31} + 5x_{32}$$

$$\text{Subject to} \qquad 5x_{11} + 2x_{12} + 6x_{21} + 5x_{22} + 7x_{23} + 2x_{31} + 8x_{32} \leq 20$$

$$x_{11} + x_{12} = 1$$

$$x_{21} + x_{22} + x_{23} = 1$$

$$x_{31} + x_{32} = 1$$

$$x_{ij} = 0 \text{ or } 1$$

The above model is referred to as zero-one integer programming model in the optimization literature. It can be solved using special techniques. The solution will give the optimal x_{ij} values that will minimize c_a. For our example, the optimal solution is:

$$x_{11}=0,\ x_{12}=1,\ x_{21}=1,\ x_{22}=0,\ x_{23}=0,\ x_{31}=1,\ x_{32}=0 \text{ and } c_a=13.$$

The solution indicates that the first component should use process 2, whereas second and third components should use process 1. The overall cost is equal to 13.

While Ostwald and Huang's model, brings a different perspective to the tolerance allocation problem, its major drawback is the use of zero-one variables (x_{ij}'s). Zero-one techniques provide solutions within reasonable computer time for small problems. However, as the number of components in an assembly increase, the required computer time increases exponentially. Hence, for complicated assemblies where several tolerance related constraints are needed, Ostwald and Huang's model may not be so attractive.

Ostwald and Huang's model is centered around the task of optimizing tolerance allocation and process selection simultaneously. Several other studies have used different approaches in trying to solve the tolerance allocation/process selection problem. Chase and Greenwood (1990), for example, have presented three approaches for the optimal simultaneous selection of both processes and tolerances while minimizing production cost: the **exhaustive search** technique, the **univariate search** technique, and the **sequential quadratic programming** (SQP) technique.

Simply, the **exhaustive search** technique looks at and compares all possible combinations of processes and optimal tolerances for all components and selects the set of processes and tolerances that provides the minimum cost of production. Since it looks at all possible solutions, the exhaustive search method is guaranteed to find the overall, true minimum (i.e., the *global* minimum) cost of production. However, searching through all possible solutions also means that the time necessary to find the answer is very high and increases tremendously as the number of possible alternatives increases.

The second of Chase and Greenwood's approaches, the **univariate search** technique, attempts to improve on the exhaustive search concept by decreasing the number of alternatives that have to be analyzed. The essence of the univariate search method is that component parts are dealt with <u>one at a time</u>. One component-part at a time, all the possible processes are looked at and the process which yields the minimum production cost for that part is found having fixed the processes for the other components. Fixing the process for that particular component part to be its minimum cost process, the method then moves on to the next part. Eventually, one by one, each part's minimum cost process is found. Finally, when all the minimum cost processes are found, tolerances are allocated. Thus, in "optimizing" the process for only one component part at a time, the number of alternatives searched is significantly reduced.

However, since the component tolerances and cost-curves may be interdependent but have been considered separately, the univariate search method is not guaranteed to find the global optimal solution. If the univariate search is run several times, though, and each time you begin with the "best" solution which was found on the last run, the global optimal solution is usually located/approximated--which is still a large reduction in the number of searches made.

The third and final technique, the **sequential quadratic programming** (**SQP**) technique, is an adaptation of a similarly named nonlinear programming technique. Generally, the SQP technique looks at the "slope" (the gradient) of the manufacturing cost function to find the minimum manufacturing cost. For example, if an initial set of processes are chosen, and the corresponding cost is used to begin the search, the SQP technique looks at the "slope" of the cost function "around" the initial cost-solution and attempts to "move" along the cost function in the direction of decreasing costs. However, even though it was able to handle complex tolerance relationships, the SQP method could not guarantee finding the global minimum manufacturing cost. This is due to the fact that a manufacturing-cost function which incorporates process selection is likely to have several or many "kinks" in it--thus producing several or many false minimum costs that "trap" the SQP algorithm (and other gradient-based techniques like it) into finding only a local (pseudo) minimum rather than the desired global minimum cost.

Acknowledging the problem with techniques similar to the SQP method, Lee and Woo (1989), have attempted to improve upon the basic premise behind the exhaustive and univariate search methods by further decreasing the number of searches made. Lee and Woo have employed what is termed a "branch and bound" procedure to accomplish their goal. Essentially, branch and bound techniques are efficient enumeration techniques that can find the global minimum of integer problems (i.e., problems with 0-1 coefficients) while greatly reduce the number of searches made. The technique was successful in lessening the number of searches made but the that number was still quite high.

Overall, it seems that two main problems occur when trying to simultaneously optimize process selection and tolerance allocation: (1) the solution techniques may take a large amount of time to solve as the problem grows larger, and/or (2) the technique may be susceptible to becoming "trapped" by a local optimum--and thus not able to find the global optimum.

Additional attempts at overcoming the aforementioned difficulties were made by Zhang and Wang (1993) and Cagan and Kurfess (1992). Both studies demonstrated application of the **simulated annealing** (SA) optimization technique to the tolerance allocation problem. The previous techniques compared alternative solutions, trying to find alternatives that would decrease the cost further. Thus, when a new, lower cost alternative was located it was made the "current best solution" and other alternative solutions were compared to it. When no other alternative solutions could be found that would further decrease the cost, the techniques stopped at the current best solution (local minimum). The SA technique , however, can temporarily accept "worse" solutions with a certain probability that decreases as the technique proceeds. Eventually, the probability of accepting a "worse" solution decreases to zero and the SA technique is forced to converge to the nearest local minimum.

Thus, since SA can accept worse solutions and "jump" to what normally may be unreachable local minima, the chances of the local minimum found by SA being better than the local minimum found by the previous techniques is quite high. Also, the chance of the SA local solution being the global solution is greater than that of the other techniques. Both Zhang and Wang (1993) and Cagan and Kurfess (1992) found that the above benefits did prove true in the tolerance allocation/process selection problem. The SA solutions were consistently better than those found by the other techniques. In fact, SA was able to solve some problems that were unsolvable by SQP and was also able to find the global optimum a good part of the time. Hence, other than the fact that it is more difficult to apply than the other methods, SA seems to be an effective technique for solving the tolerance allocation/process selection problem.

Wu et al (1988) and Chase and Greenwood (1988) discuss two other tolerance allocation methods. These methods are the proportional scaling method and the constant precision methods. Both methods are <u>not</u> based on mathematical optimization models. In the proportional scaling method tolerances are split up into design tolerances and fixed tolerances. Fixed tolerances are subtracted from the assembly tolerance to determine maximum allowable design tolerance. If the assembly design tolerance exceeds this value, the design tolerance associated with each component is scaled down using a scaling factor p. Let t_{fi} and t_{di} denote fixed and design tolerances for component i. Similarly, define t_{fa} and t_{da} for the assembly tolerance. Then,

$$t_i = t_{fi} + t_{di}$$

$$t_a = t_{fa} + t_{da}$$

If $\sum_{i=1}^{n} t_{di} > t_{da}$, then $p = \dfrac{t_{da}}{\sum_{i=1}^{n} t_{di}}$ and component design tolerances are scaled to pt_{di}. The method aims at adjusting the component tolerances to meet the required assembly tolerance. The constant precision factor method assumes that as the component dimension is increased, the tolerance on the component increases with the cube root of the dimension. Hence, tolerances are calculated using

$$t_i = p(d_i^{1/3})$$

where $p = \dfrac{t_a}{(\sum_{i=1}^{n} d_i^{2/3})^{0.5}}$, and d_i = basic dimension of the part.

Peters (1970) reviews some of the tolerance analysis and tolerance allocation methodologies that we have described earlier. He also demonstrates a previously devised method based on a two-component assembly. Since the mathematical solution for larger assemblies is too involved, the technique is omitted from the paper.

In the last decade, Parkinson (1982) has contributed some interesting approaches to the tolerance literature. He develops a technique to estimate failure probability in reliability analysis. Parkinson's technique is applied to problems associated with the tolerancing of the dimensions of manufactured components. A procedure is described which permits, for the general nonlinear problem, the deduction of the estimate of the frequency with which a set of components will fail to assemble together according to the design specification due to inevitable variations in their dimensions.

The method may be used at the design stage to adjust the relative size of tolerances on different dimensions and to permit the relaxation of the tolerances to the maximum degree commensurate with a required level of assurance of correct assembly. Since the methods relate such a rejection risk to the statistics of the component dimensions, these methods permit the rational selection of part dimensions and tolerances for a given acceptable risk level.

Parkinson (1985) develops a group of computer programs in FORTRAN. One set of these programs enables the adjustment of tolerances to minimize the risk of rejection or malfunction on assembly. Alternatively, given certain relative cost data, optimization of the tolerances and/or dimensions is carried out based on the minimization of overall cost. In order to explain the theory of dimensioning/tolerances and risk computation, he also provides an example concerning tolerance optimization.

The program is divided into two subprograms: risk assessment and tolerance optimization. These subprograms allow both rapid assessment of tolerances on a set of component dimensions by indicating the magnitude of risks of unsatisfactory assembly by any particular mode, the overall or joint risk of rejection of the assembly, and the optimization of tolerances to minimize overall cost of manufacture given accuracy and assembly including cost of rejection.

The nominal values of design variables are of interest in conventional optimization. Michael and Siddall (1981) treat tolerances as design variables because the optimization problem includes the optimal allocation of manufacturing tolerances. This paper limits the model to a production process with 100 percent acceptability.

Acceptability can be defined as a fraction of the components satisfying manufacturing specifications. The method used in this paper might be applicable to either job or batch type production where there is less likelihood of scrapping components which do not meet design specifications.

Unlike a conventional optimization problem where a single point is of interest, the optimization scheme in this paper creates a region of interest. If the constraint region satisfies the parallel convexity assumption, then all of the points inside and on the boundaries of the tolerable optimization region will determine optimal feasible region. The paper states that the optimal tolerance range increases significantly when the design variables associated with the tolerances are allowed to be unsymmetrically allocated.

In an attempt to bridge the gap between design and production engineers, Michael and Siddall (1982) use nonlinear optimization for the optimal tolerance assignment. The upper and lower limits of the random variables of an engineering system are allocated so as to minimize production cost with an allowance provided for the system scrap.

If full acceptance is not required and a few scrap designs are permitted to occur, the tolerance can be increased, and more economical overall design can be achieved. This paper proposes a working procedure for optimal allocation of tolerances for designs which require less than full acceptance.

Conclusions that can be drawn from review of the tolerance analysis and tolerance allocation literature can be summarized as follows:

(1) Statistical analysis of the tolerance problems can only be done when tolerance distribution functions are applicable. This is not usually the case in real life applications.

(2) Nonstatistical analysis provide practical but less accurate tolerance analysis.

(3) Cost functions which are required for optimization methods can not be easily acquired. Exponential cost function is normally assumed.

(4) Nonlinear tolerance allocation models are more realistic but harder to solve for real life problems.

3. DEVELOPMENT OF A PROTOTYPE SYSTEM FOR TOLERANCE ALLOCATION AND ANALYSIS

3.1 Tolerance Allocation

It is a known fact that tolerances to dimensions of components or assembly are assigned based on design standards (ANSI or ISO), trade group guidelines, company design manuals, government regulations or customer/user feedback. They do not directly incorporate process capability of manufacturing process and cost of producing these tolerances on the particular dimensions. Hence, a unique methodology has been developed for allocating tolerance to each dimension with the objective of minimizing manufacturing cost, subject to the constraint of functional (assembly) requirement.

The optimal tolerance allocation methodology uses the following concepts:

1. Interaction with the User
2. Process Cost-Tolerance function
3. Tolerance allocation methods
4. Expert system

Each of these concepts are individually detailed before discussing the overall system.

<u>Interaction with the User</u>

The expert system interacts with the user to obtain information as to whether the allocation has to be carried out on radial dimensions or linear dimensions. If allocation is on the radial dimensions, the user has to provide information to the expert system as to the nominal dimension and the type and class of fit desired. If allocation is on the linear dimensions, the user has to provide information as to the type of fit desired on each dimension and the classification of the component (whether it is internally stepped or externally stepped and the ratio of the length to diameter of the component). Based on the information provided an assembly function is developed for use in tolerance allocation.

Process Cost-Tolerance Function

Many mathematical functions have been proposed to fit manufacturing cost-tolerance field data. Five cost-tolerance functions have been discussed in literature (Wu et al, 1988):

1. Sutherland function
2. Reciprocal square function
3. Reciprocal function
4. Exponential function
5. Michael-Siddall function

In order to make better use of empirical production data compiled by Trucks (1974), the following exponential function is used in our software:

Cost $c_i = a_i \exp(-b_i t_i)$ ....(1)

where

t_i = tolerance of dimension i

a_i, b_i= Cost-tolerance parameters for dimension i.

Each tolerance dimension has a different cost value depending on the process used to produce it. The process parameters are associated with tolerances using the feature specified by the tolerance since a feature is always related to the process used to produce it. In this work, since only rotational parts are considered, the mechanical features are restricted into two basic feature types which are external rotational surface features, hole features and plane features. Based on the information provided by the user a search to match the feature category for the identified feature is then conducted. When completed, the relation between the production cost-tolerance model M_j and the design tolerance t_i is found. This is demonstrated in Figure 3.1 (Dong and Soom, 1989). Production cost-tolerance relations and the process-capability data of the two feature types are based on empirical data compiled by Trucks (1974). Empirical cost-tolerance relations of production operations including rough turning, semi-finish turning, cylindrical grinding, drilling, boring, honing, and internal grinding have been utilized. These are plotted in Figure 3.2.

Cost-tolerance parameters of equation 1 were obtained by linear regression (equations 2 and 3).

$$\ln(a_i) = [\Sigma \ln(c_i)-b_i\Sigma t_i] / n \quad \text{.................................(2)}$$

$$b_i = [-n\Sigma t_i \ln(c_i)+\Sigma t_i \Sigma \ln(c_i)] / C \quad \text{.................................(3)}$$

where

$C = n\ [\Sigma(t_i^2) + (\Sigma t_i^2)\ ]$

n = number of data points

Cost parameters a_i and b_i for external rotational surface features is shown in Table 3.1. Similarly in the case for hole features it is shown in Table 3.2. In the case of plane features, it is classified into 4 categories:

(1) External step

(2) Internal step

(3) cylindrical Facing with low length to diameter ratio

(4) Cylindrical Facing with high length to diameter ratio

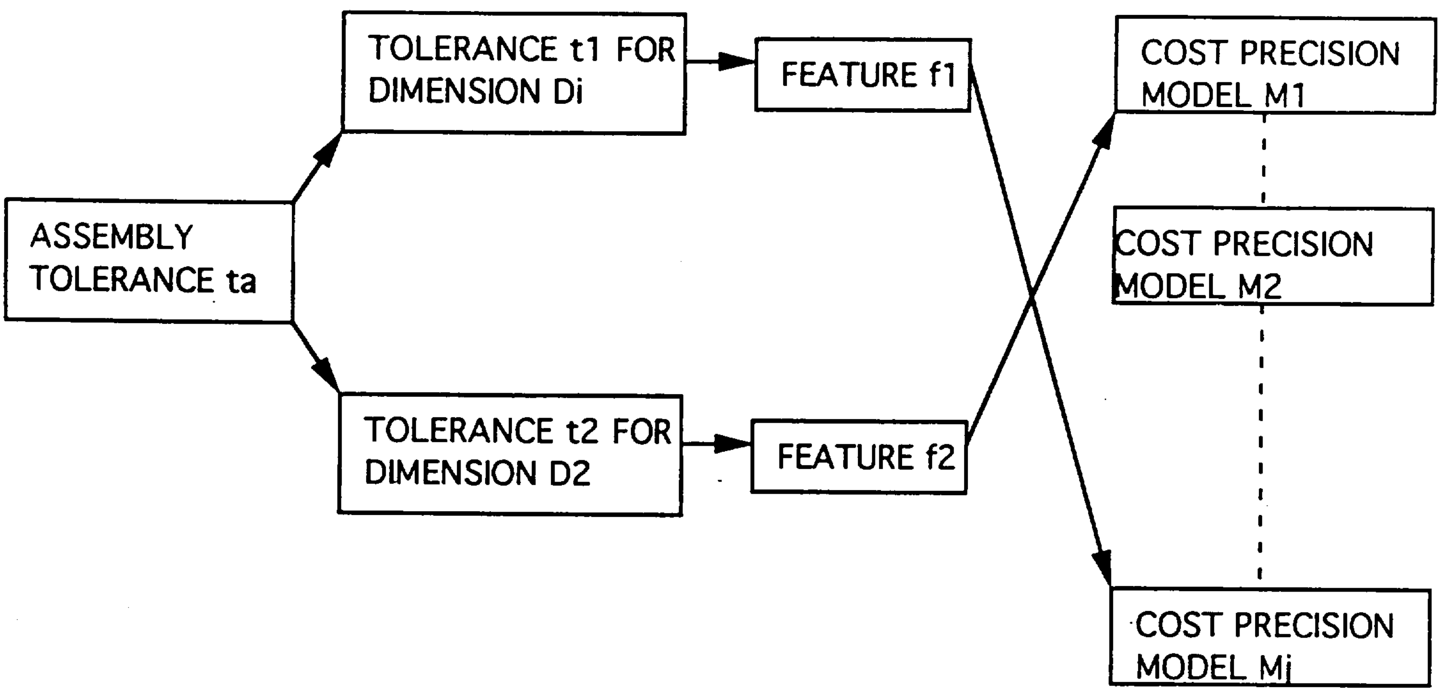


Figure 3.1 Relationship among design features and production cost-precision models (from Dong and Soom, 1989)

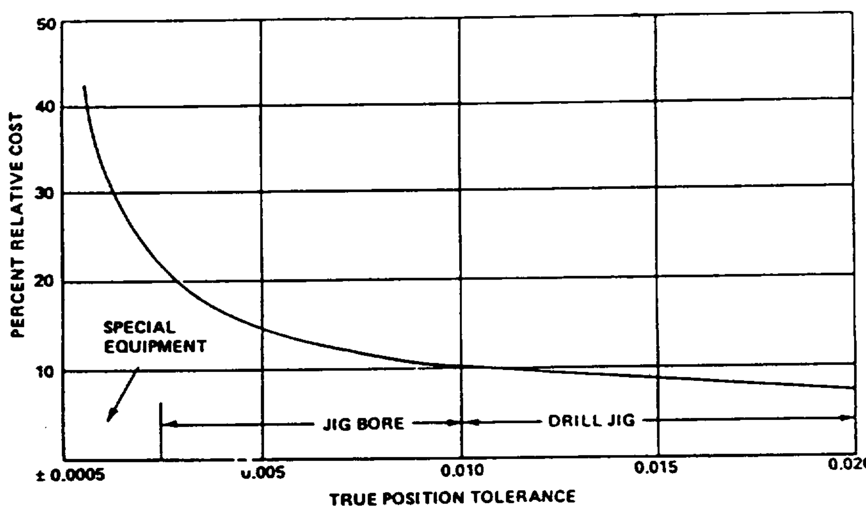


Figure 3.2 Cost vs true position tolerance (from Trucks, 1974)

Table 3.1 Model Mj - Cost-tolerance parameters/Process-capability for external rotational surface features

Operation	Minimum Tolerance	Maximum Tolerance	a	b
Rough_turn	0.004	0.05	31.6	28.6
Semi_turn	0.002	0.004	31.6	28.6
Finish_turn	0.00075	0.002	31.6	28.6
Grinding	0.00025	0.00075	84.0	80.9
Honing	0.0001	0.00025	84.0	80.9

Table 3.2 Model Mj - Cost-tolerance parameters/Process-capability for hole features

Operation	Minimum Tolerance	Maximum Tolerance	a	b
Drilling	0.01	0.05	10.67	8.38
Reaming	0.005	0.015	10.67	8.38
Boring	0.00075	0.005	10.67	8.38
Internal Grindin	0.00025	0.00075	136.59	124.72
Honing	0.0001	0.00023	136.59	124.72

All these categories are shown in Figures 3.3a-3.3d. Production processes considered for each category is shown in Tables 3.3 to 3.5. Also since the process-capability data is dependent on the range of dimension for which the tolerance is to be allocated, a series of such tables are created with different process-capability data for each range of dimension without altering the cost-parameters.

The curves generated from the corresponding exponential equation 1 are shown in Figure 3.4. They provide a good match to the empirical curves. Hence, these cost-tolerance models M_j were stored in the form of database files. The data set is not always complete and some are as old as thirty years. All the same they provide quantitative measures of relative production costs of different processes for the development of the methodology. Also the process capability data has to be tailored to the resources available in a specific shop and to the specific shop policies.

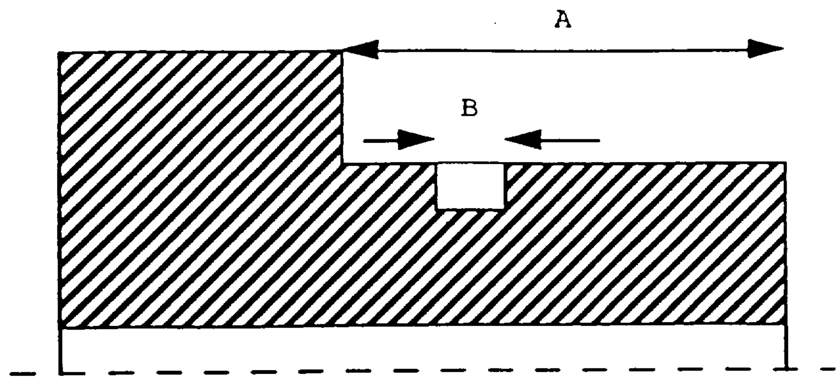


Figure 3.3a Plane features: External step or groove

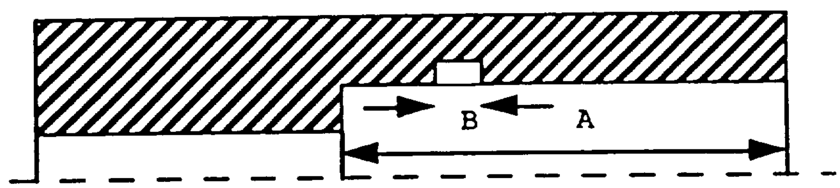


Figure 3.3b Plane features: Internal step or groove

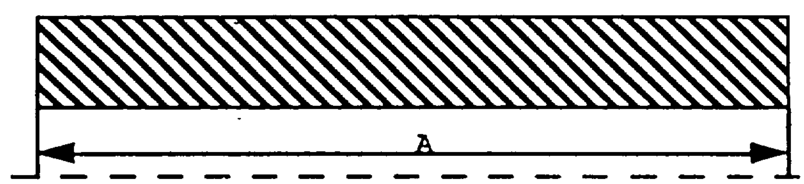


Figure 3.3c Plane features: Cylindrical facing with high length to diameter ratio

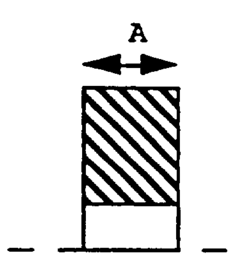


Figure 3.3d Plane features: Cylindrical facing with low length to diameter ratio

Table 3.3 Model Mj - Cost-tolerance parameters/Process-capability for plane features: External step or groove

Machining Process	Minimum Tolerance	Maximum Tolerance	a	b
Rough_turn	0.004	0.05	31.6	28.6
Semi_turn	0.002	0.004	31.6	28.6
Finish_turn	0.00075	0.002	31.6	28.6
Grinding	0.00025	0.00075	84.0	80.9
Honing	0.0001	0.00025	84.0	80.9

Table 3.4 Model Mj - Cost-tolerance parameters/Process-capability for plane features: Internal step or groove

Machining Process	Minimum Tolerance	Maximum Tolerance	a	b
Rough_ Bore	0.01	0.05	10.67	8.38
Semifin_Bore	0.0025	0.01	10.67	8.38
Finish_Bore	0.00075	0.0025	10.67	8.38

Table 3.5 Model Mj - Cost-tolerance parameters/Process-capability for plane features: Cylindrical facing low length to diameter ratio

Machining Process	Minimum Tolerance	Maximum Tolerance	a	b
Rough_turn	0.005	0.03	31.6	28.6
Milling	0.002	0.004	39.8	19.2
Rotary_Grind	0.0002	0.001	84.0	80.9
Lapping	0.00015	0.00030	84.0	80.9

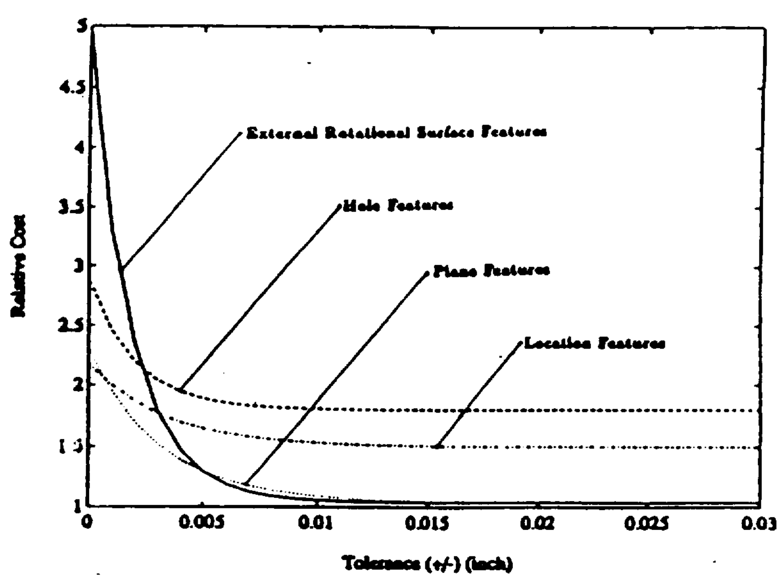


Figure 3.4 Modeled production cost-tolerance relations for basic features (from Dong and Soom, 1989)

Tolerance Allocation Methods

Six tolerance allocation techniques have been cited in the literature. These methods are:

1. Proportional scaling method
2. Constant precision factor method
3. Lagrange multiplier method
4. Geometric programming method
5. Linear programming method (Worst-case model)
6. Non-linear programming method (Statistical model)

The proportional scaling method (Chase and Greenwood, 1988) starts with the designer allocating tolerances to components using the database. If the sum of component tolerances is within the specified assembly tolerance, then they are used. Otherwise, each component tolerance is scaled relative to its magnitude.

The constant precision factor method (Chase and Greenwood, 1988) is similar to the proportional scaling method and assumes that tolerances increases as the cube root of component size. That is,

$t_i = p(d_i)^{1/3}$ where

t_i = component tolerance

d_i = component dimension

p = precision factor

The precision factor is calculated such that the tolerance build up will not exceed the specified value.

The Lagrangean multiplier method (Wu et al, 1988) is an optimization method of tolerance allocation where the cost-tolerance function could be Sutherland, reciprocal, square reciprocal or exponential function. The formula for this method reduces to:

$t_i = -(1/b_i)\ln[\exp(A)/(a_i b_i)]$

where

$A = \{\Sigma\ [\ln(a_i b_i)/b_i] - t_a\}/C$

$C = \Sigma(1/bi)$

t_a = specified assembly tolerance

The Geometric programming method (Wu et al, 1988) is only adoptable for the exponential cost-tolerance function. The component tolerances are determined using the following equations:

$t_i = \{t_a/A + \ln[(a_i b_i G)/R]\}/b_i$ where

$A = \Sigma(1/b_i)$

$R = \prod\ (a_i)^{1/(biA)}$

$G = \prod\ (1/bi)^{1/(biA)}$

Linear programming method (Wu et al, 1988) is also computerized for problems where the cost-tolerance function can be linearly approximated. The model is given as:

Minimize $\Sigma c_i t_i + A'$

s.t. $\Sigma\ t_i <= t_a$ and $t_i >= 0$ where

c_i = slope of i'th linearized cost-tolerance line

A' = constant

Non-linear programming method is computerized using Box-Jenkins method for non-linear objective function and non-linear constraint. The model is given as:

Minimize Cost = $\Sigma a_i \exp(-b_i t_i)$

s.t. $\Sigma t_i^2 <= t_a^2$ and $t_i >= 0$

It was decided that for the tolerance allocation model only Linear programming and Nonlinear programming methods would be used. The reasoning was that the first two techniques, Proportional scaling method and Constant Precision factor method, were subjective and based on experience and hence, may not be optimum with respect to manufacturing cost. Geometric programming method, Lagrange multiplier method and Linear programming method would give a similar result for the existing problem domain, and hence, linear programming is chosen for linear constraints and linearized exponential cost-tolerance function as the objective function. Nonlinear programming method is chosen for nonlinear constraints and exponential cost-tolerance function as the objective function.

Expert System

A rule based expert system is used for allocating optimal tolerances to the component dimensions. Expert system software used is VP-Expert whose choice is justified by its interfacing capabilities with the user, database and spreadsheet software, and programming languages like QuickBasic, FORTRAN etc. and its chaining facility with different knowledge base program.

This approach seems appropriate since the process data association is of the deductive and reasoning type, whereas tolerance calculation is numerical. Therefore, an expert system with a numerical routine interface provides useful capabilities not offered by a purely numerical approach. Another advantage of expert system is related to the fact that manufacturing knowledge and production data can be comprised of dynamic information. Changes in production environment may result not only in new cost-precision data, but also new rules or logic to perform tolerance design.

The Overall Tolerance Allocation System

Figure 3.5 shows the interfacing of tolerance allocation programs and database files with the expert system or optimal tolerance allocation knowledge base system (KBS). For design tolerance allocation the KBS interacts with the user to obtain the dimensions of the component for which tolerances have to be allocated and the functional/assembly requirement in the form of type of fit and class of fit. Based on the user's response, KBS automatically accesses the ANSI standards for retrieving the appropriate design tolerance values.

For optimal tolerance allocation, the KBS first accesses the ANSI standards for retrieving the initial tolerance values and calculates the assembly tolerance value. As per the dimension value, the KBS accesses the appropriate cost-tolerance model M_j (as shown in Tables 3.1 - 3.5) and using the initial design tolerance value retrieves the appropriate cost-tolerance parameters. These parameters and the assembly tolerance value are then supplied to the tolerance allocation programs by the KBS. The optimization programs allocate tolerances as per the worst case or the statistical model (linear or non-linear programming methods) and these values of the tolerances are displayed to the user. The user has an option to choose the method which he/she feels comfortable with and update drawings with the relevant tolerance details.

The optimal tolerance allocation module allocates equal bilateral tolerances on all the components. However the design tolerances as per the ANSI standards are not necessarily equal bilateral. Therefore after the optimal tolerances are determined, it is necessary to adjust the basic dimension of each part to maintain the required assembly function.

As an example consider an RC_1 radial running fit on dimension 0.2 inches. The design tolerances as per the ANSI standards are:

$$\text{Hole : } 0.2\begin{matrix}+0.0002\\-0.0000\end{matrix} \qquad \text{Shaft : } 0.2\begin{matrix}-0.00015\\-0.0003\end{matrix}$$

Converting this into equal bilateral tolerances would require the basic dimension to be modified as given below:

$$\text{Hole : } 0.2001\begin{matrix}+0.0001\\-0.0001\end{matrix} \qquad \text{Shaft : } 0.199975\begin{matrix}+0.000075\\-0.000075\end{matrix}$$

Therefore after the optimal allocation of tolerances, the basic dimension of the parts are adjusted automatically as shown above.

The detailed procedure is shown in the form of a flow chart in Figure 3.6.

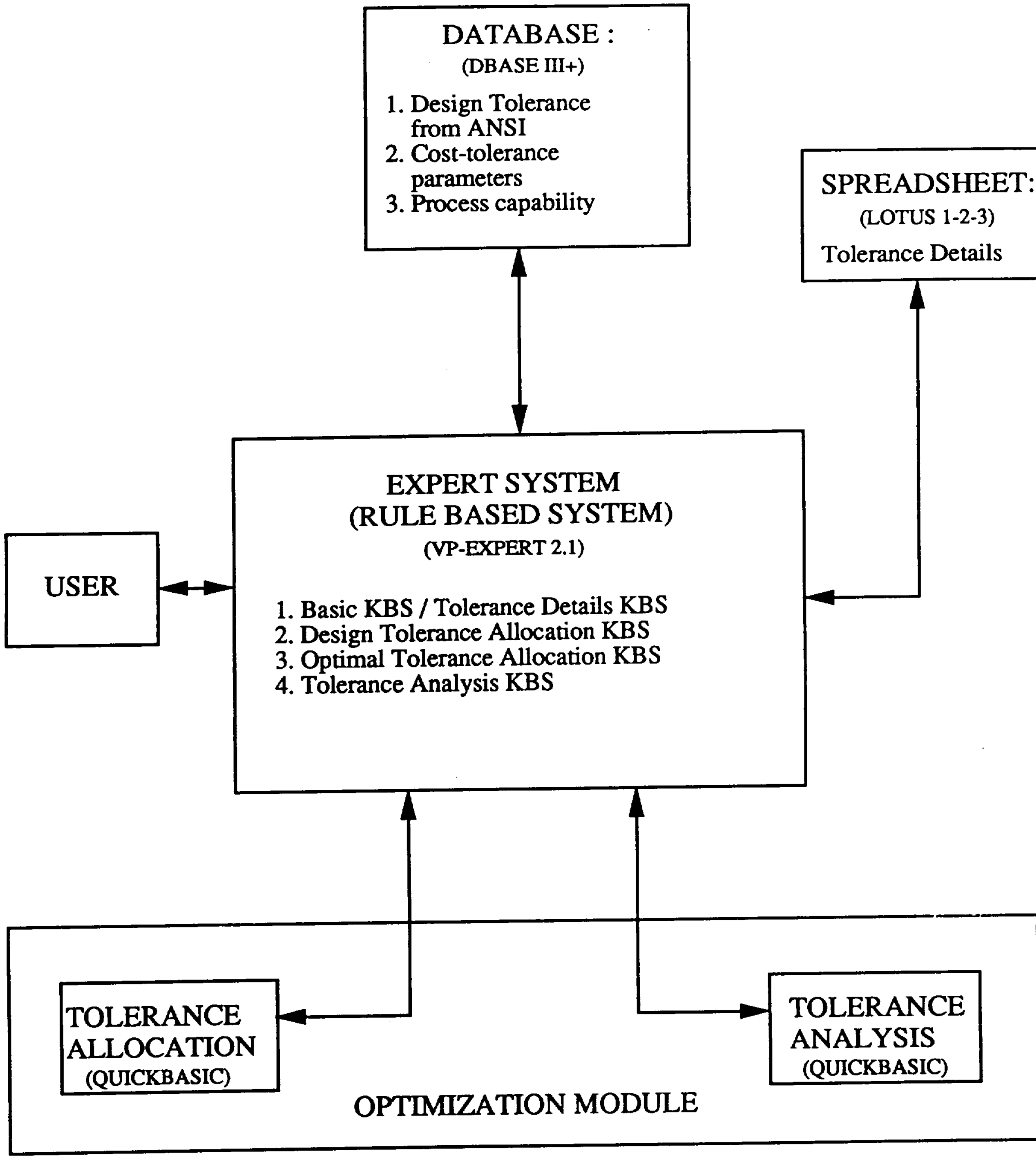
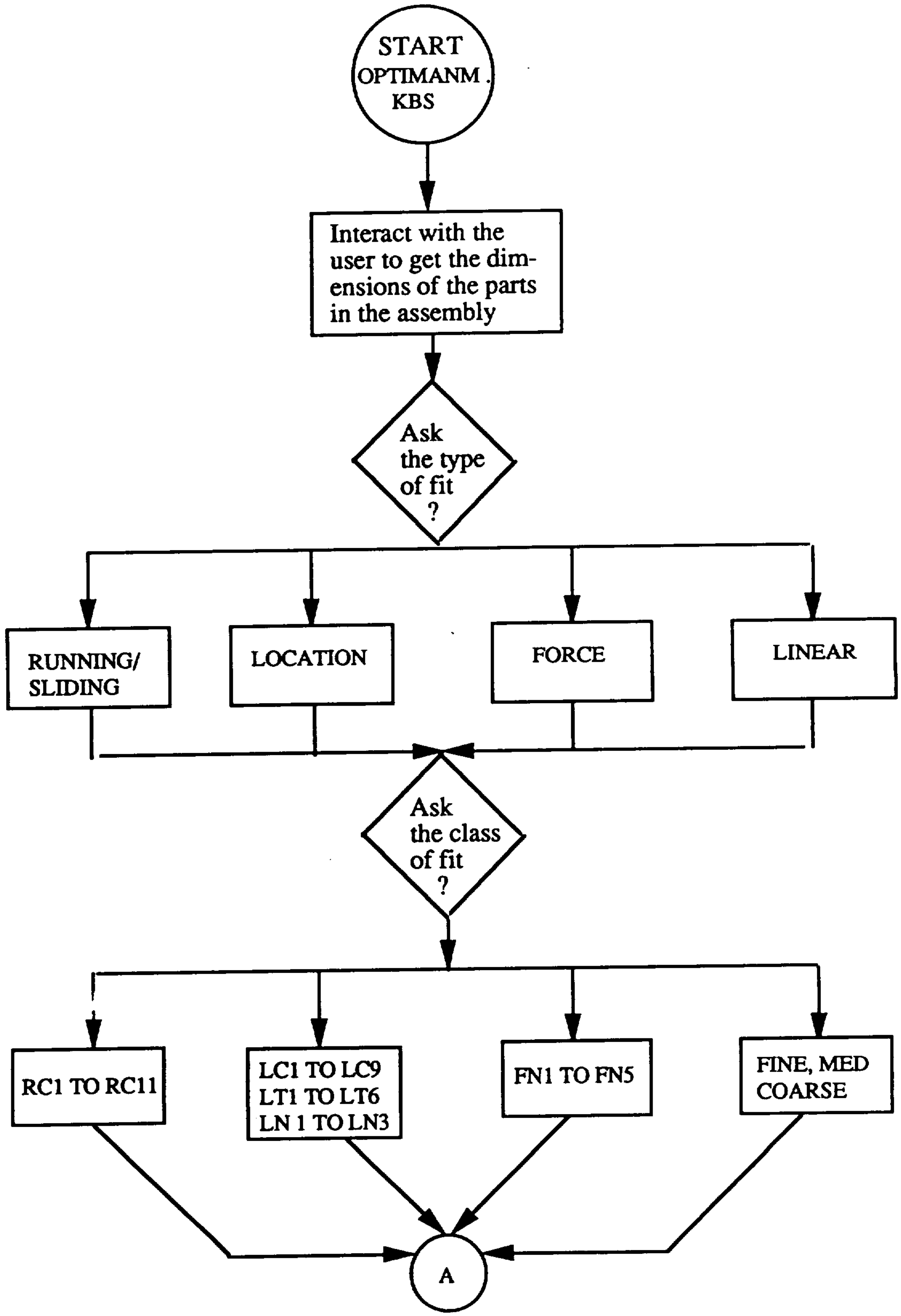


Figure 3.5 Basic Framework of the System


START
OPTIMANM.
KBS
Interact with the user to get the dimensions of the parts in the assembly
Ask the type of fit ?
RUNNING/ SLIDING
LOCATION
FORCE
LINEAR
Ask the class of fit ?
RC1 TO RC11
LC1 TO LC9
LT1 TO LT6
LN 1 TO LN3
FN1 TO FN5
FINE, MED
COARSE
A

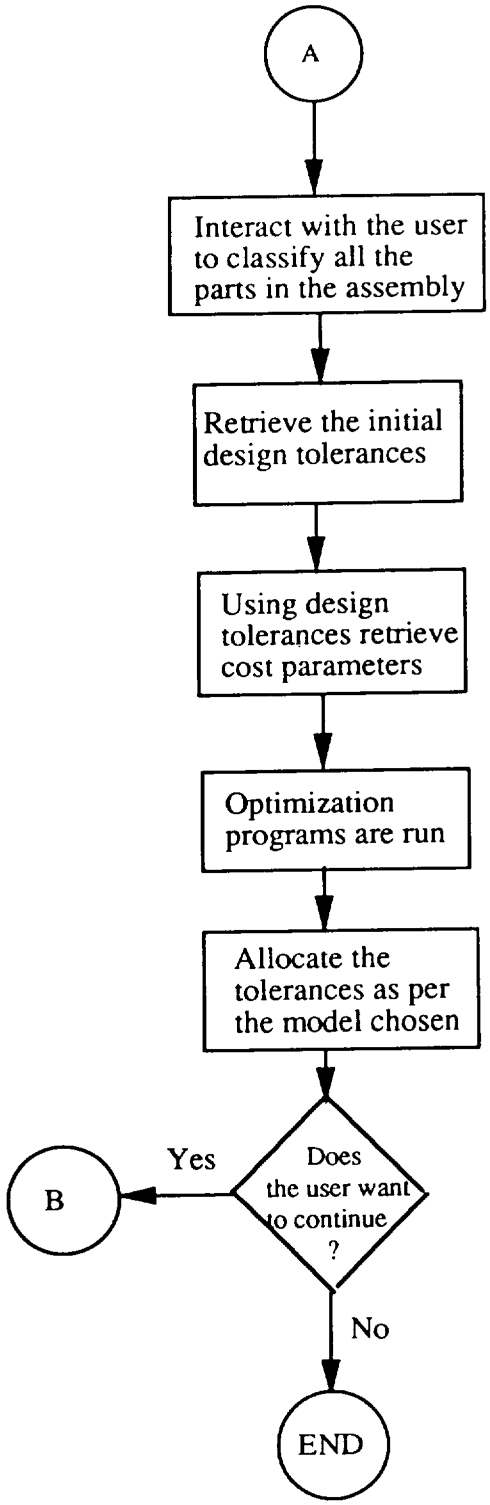


Figure 3.6 Flow Chart: Optimal Tolerance Allocation (OPTIMANM.KBS)

3.2 Tolerance Analysis

In the tolerance allocation model, it is presumed that the designer has the control to allocate tolerances to the component dimensions, and there is no external constraint. But in cases where the components are purchased (from external vendors), tolerances to component dimensions have already been assigned. In such a case, the designer's duty is to ensure that critical clearances/interferences are maintained and that tolerance stack-up would not result in unexpected assembly rejections. Hence, a tolerance analysis procedure has been developed to predict assembly tolerance of a tolerance chain, given component tolerances.

The Tolerance Analysis Model

The model uses the following concepts:

1. Interaction with the user
2. Tolerance Analysis methods
3. Expert System

Each of these concepts are detailed individually. The overall analysis concept is discussed in the end.

Interaction with the User

The KBS interacts with the user to obtain the tolerance for each dimension that make up the assembly. This will be used to establish the assembly tolerance by tolerance analysis programs.

Tolerance Analysis Methods

Eight tolerance analysis methods have been cited in the literature and the first six have been programmed using QuickBasic. These methods are:

1. Worst-case model

2. Statistical model

3. Spott's modified model

4. Modified statistical model

5. Mean shift model

6. Moment model

7. Monte-Carlo model

8. Hybrid model

The worst-case model (Wu et al, 1988) is represented by:

$T = \Sigma\, t_i$ where

T = calculated assembly tolerance

t_i = individual component tolerances

This model can guarantee satisfaction of the specified assembly tolerance with 100% probability, resulting in a very large value of the resulting (calculated) assembly tolerance.

The statistical model (Wu et al, 1988) for normal distributions, is given by:

$T = (\Sigma\, t_i^2)^{0.5}$

The assembly tolerance calculated using this model is the smallest, in most cases, compared with all analysis models, especially when the number of components is large.

Spott's modified model (Wu et al, 1988) for tolerance is given by:

$T = 0.5(\Sigma t_i + (\Sigma\, t_i^2)^{0.5})$

This model is a combination of the worst case model and the statistical model. For skewed distributions this model determines an assembly tolerance which leads to fewer rejects.

The modified statistical model (Wu et al, 1988) is given by the following equation:

$T = c(\Sigma t_i^2)^{0.5}$ where

c = correction factor, typical value 1.4 or 1.5

This model introduces a correction factor c which is to account for non-random factors such as errors in predicting component tolerance distributions.

Mean shift model (Chase and Greenwood, 1988) is described by:

$T = \Sigma m_i t_i + (\Sigma [(1-m_i)t_i]^2)^{0.5}$ where

m_i = possible range of mean shift for the i'th component

(expressed as a function of its range).

This model is preferred when few or poorly controlled components (high process variability) define the assembly. The moment model (Wu et al, 1988) is represented as:

$T = X_{max} - X_{min}$ Xmax = M + 3D   Xmin = M - 3D

$M = \Sigma m_i$ $D = (\Sigma \sigma_i^2)^{0.5}$ where

M = assembly mean tolerance

m_i = the i'th component mean tolerance

D = standard deviation of the assembly tolerance

σ_i = standard deviation of the i'th component tolerance

X_{max} = component maximum dimension

X_{min} = component minimum dimension

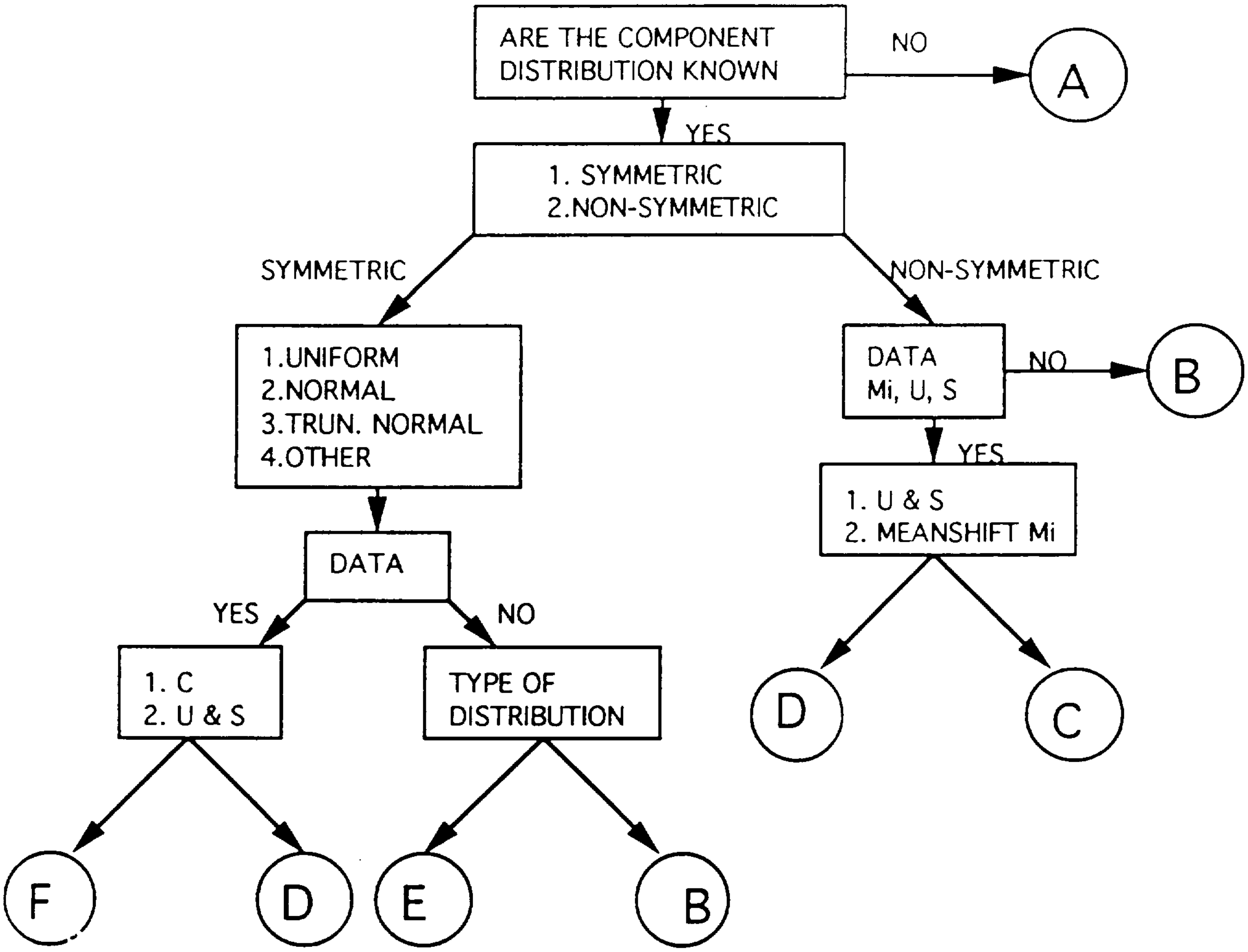
This model is preferred over the Monte-Carlo model when components are identically distributed, and the mean and standard deviations of each component tolerance distribution can be calculated.

Expert System

A rule based expert system is used for tolerance analysis of radial and linear mating dimensions of an assembly of rotational parts. An interaction with the user is possible with the use of an expert system. A decision making procedure was developed using a knowledge base systems approach to help the user select one of the tolerance analysis methods. The decision tree is shown in Figure 3.7. The user is asked questions in a VP-Expert environment on the type of component dimensional tolerance distribution and also its characteristics. According to the response given by the user and the data supplied by the user, a decision is made on the type of tolerance analysis method to be used.

The Overall Tolerance Analysis System

Figure 3.5 shows the interfacing of tolerance analysis programs, and database files with the tolerance analysis knowledge base system (KBS). KBS interacts with the user to get the dimensional tolerances on the radial and linear mating dimensions. As per the response given by the user, a decision is made on the tolerance analysis method and the corresponding tolerance analysis program is executed. The assembly tolerance is then displayed to the user. The detailed procedure is shown in the form of a flow chart in Figure 3.8. This procedure is carried out for each assembly function. The user can hence, make a decision to buy the components from a particular vendor based on these assembly tolerance values.



A. WORST CASE MODEL

B.SPOTT'S MODIFIED MODEL

C.MEAN SHIFT MODEL

D.MOMENT MODEL

E.STATISTICAL MODEL

F.MODIFIED STAT. MODEL

C: statistical constant
U: mean of the distribution
S: standard deviation of the distribution

Figure 3.7 Decision tree for tolerance analysis

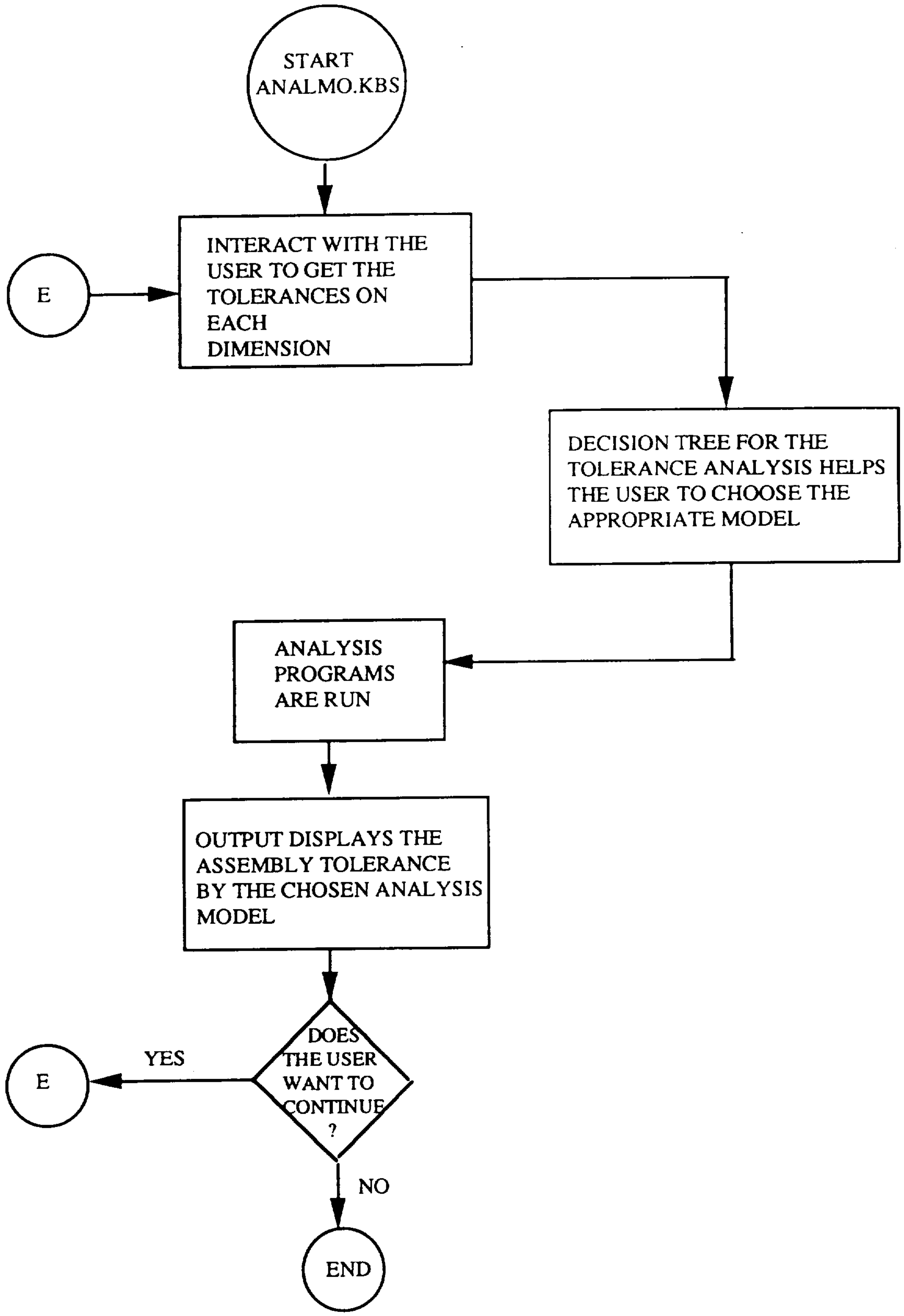


Figure 3.8 Flow Chart: Tolerance Analysis (ANALMO.KBS)

4. GUIDELINES FOR SOFTWARE USE

4.1 Introduction

The following are the instructions the user need to follow for running the "CATALL" software package.

1. Make the following three directories on the hard drive:

    a. vpx21

    b. dbase

    c. gw

2. Load the following software on the hard drive in their respective directories

    a. VP-Expert 3.0 in vpx21 directory

    b. dBASE III Plus in dbase directory

    c. QuickBasic in the gw directory

3. Run the batch file "load" by typing *load* at the B:\> or A:\> prompt (the drive in which the floppy is placed). This will load the required files in their respective directories.

4. At the C:\> prompt type *cd vpx21*

5. At the C:\VPX21> prompt type *vpx*

6. After having entered the VP-Expert software select *Consult* from the screen and press *enter*

7. Select the file *basemo.kbs*

8. Select the option *Go* from the menu

9. Proceed according to the instructions in the program

10. Exit vpx21 directory by selecting *QUIT*

4.2 Example

The tolerance allocation and tolerance analysis methodology are implemented on a Zenith SX-80386 system using dBASE III Plus, Lotus 1-2-3, VP-Expert software and QuickBasic language. The system has been tested under several applications using different rotational mating component assemblies.

After having selected the file *basemo.kbs* from the menu in the VP-Expert software the following screens describing the overall "CATALL" package appear.

A complete flexible package for tolerance allocation based on standard handbook or Optimization models has been developed for designers and manufacturers who have to allocate tolerances to dimensions in a drawing. This package incorporates manufacturing knowledge by eliminating any infeasible tolerances allocated with regards to process capability and also the manufacturing cost. Hence, it serves as a communication interface between CAD and CAM systems.

At present, this system handles only concentric rotational parts with linear and circular dimensions. This program can however, be extended to incorporate other types of dimensions involved in a drawing and prismatic/complicated parts.

This rule-based program has been divided into four major modules:

1. Basic rule-based program/Tolerance details (BASEMO.KBS)
2. Design tolerance allocation (DESIGNMO.KBS)
3. Optimal tolerance allocation (OPTIMANM.KBS)
4. Tolerance analysis (ANALMO.KBS)

This basic program links up the above modules depending upon choice.

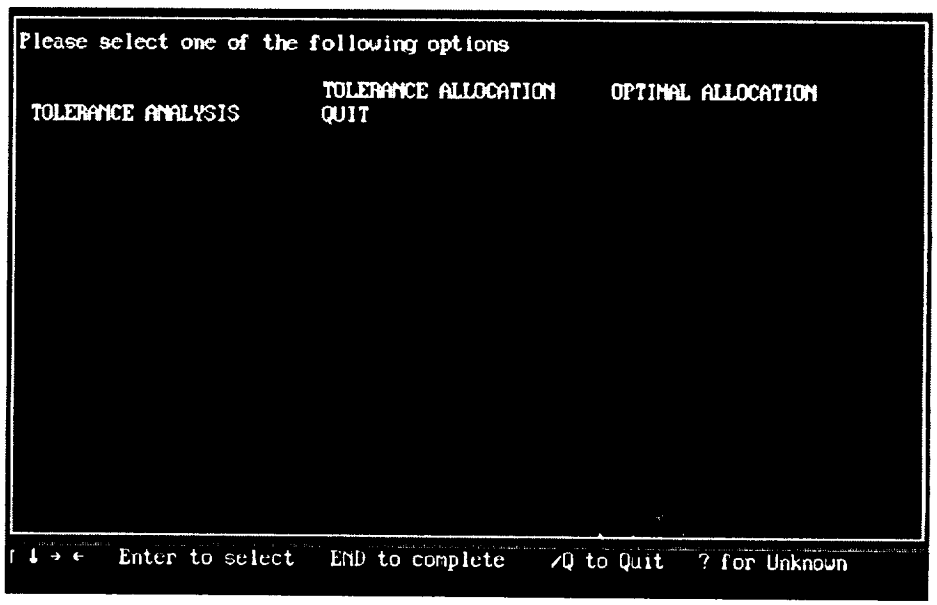
The screen below asks the user to select one of the options :

for general description on the fits select 'DESCRIPTION OF FITS',

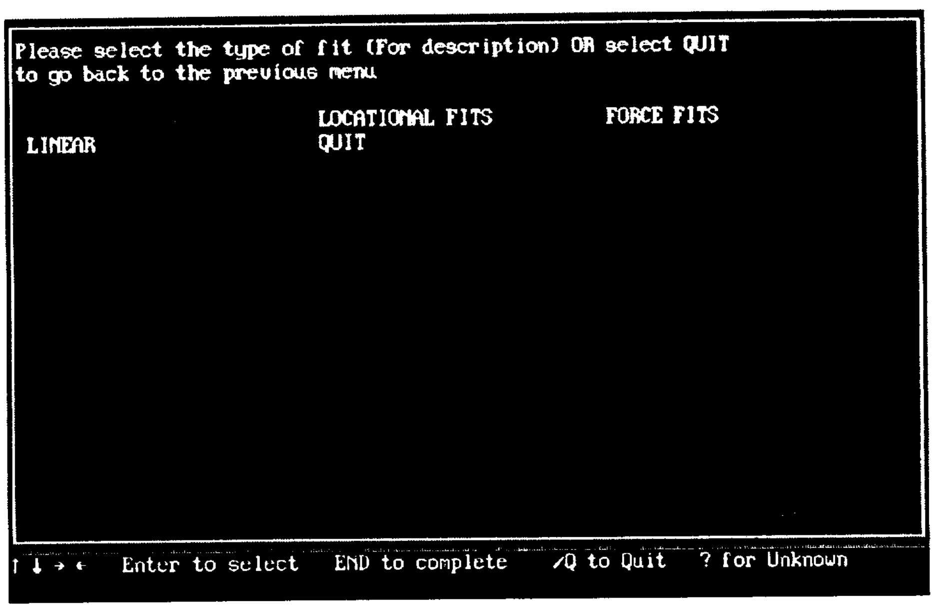
for allocation of design tolerance select 'TOLERANCE ALLOCATION',

for optimal allocation of tolerances select 'OPTIMAL ALLOCATION',

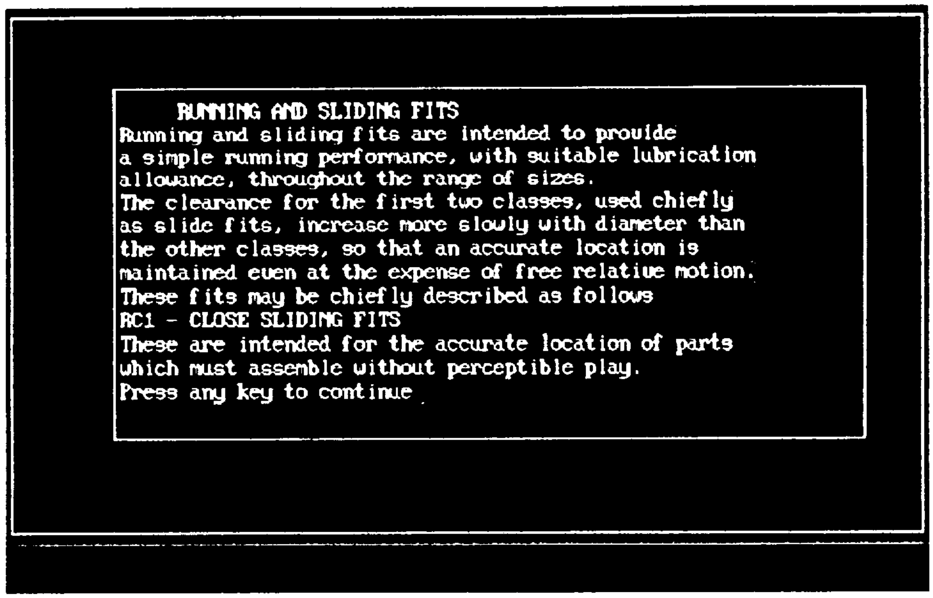
for tolerance analysis select 'TOLERANCE ANALYSIS'.

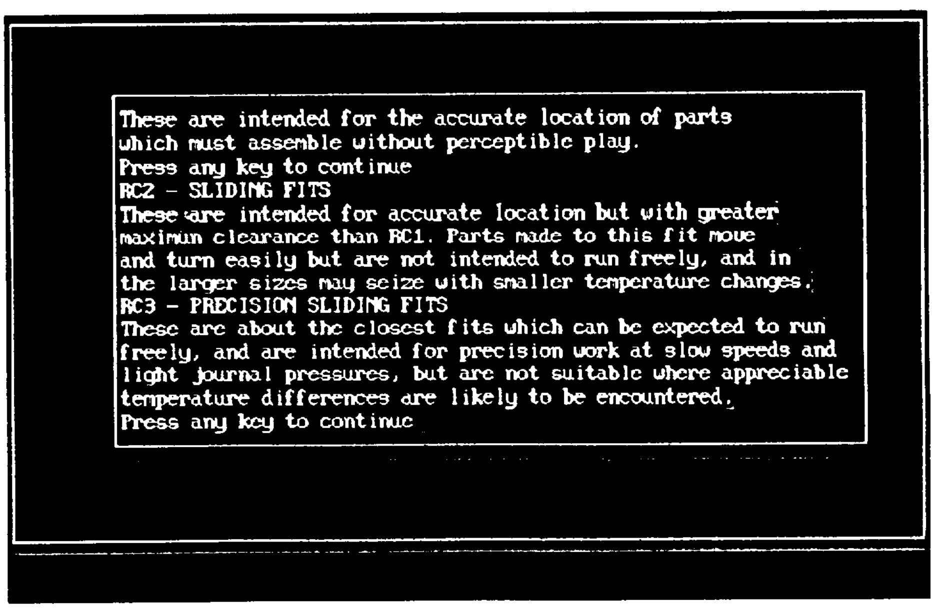


If the option 'DESCRIPTION OF FITS' is selected the following screen would appear and the user can select any one of the options to obtain information on the respective type of fits. Selecting QUIT would make the program return to the screen shown above.



The following screens give a detailed information about the Running/Sliding Fits.





where accurate location and minimum play is desired.
RC5/RC6 - MEDIUM RUNNING FITS
These fits ar intended for higher running speeds, or heavy
journal pressures or both.
RC7 - FREE RUNNING FITS
These are intended for use where accuracy is not essential,
or where large temperature variations are likely to be
encountered, or under both of these conditions.
Press any key to continue
RC8/RC9 - LOOSE RUNNING FITS
These are intended for use where wide commercial tolerances
may be necessary, together with and allowance on the external
member.
Press any key to continue

temperature differences are likely to be encountered.
Press any key to continue
RC4 - CLOSE RUNNING FITS
These are intended chiefly for running fits on accurate
machinery with moderate surface speeds and journal pressures,
where accurate location and minimum play is desired.
RC5/RC6 - MEDIUM RUNNING FITS
These fits ar intended for higher running speeds, or heavy
journal pressures or both.
RC7 - FREE RUNNING FITS
These are intended for use where accuracy is not essential,
or where large temperature variations are likely to be
encountered, or under both of these conditions.
Press any key to continue

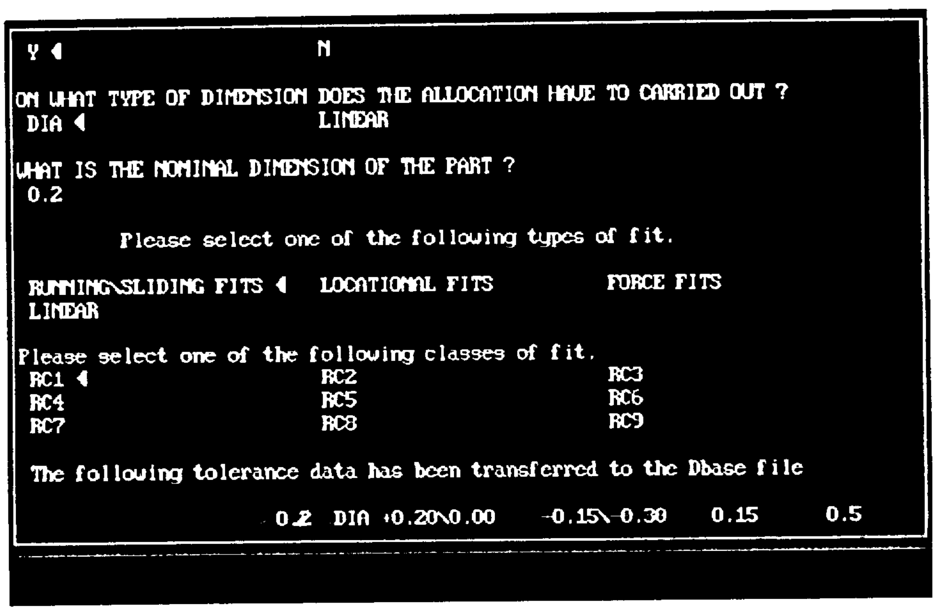
If the option 'TOLERANCE ALLOCATION' is selected, the following two screens giving the general information about the design tolerance allocation module appears.

```
This expert system retrieves tolerances for rotational parts from a database.

There are five standard classes of diametrical fits used in the
UNITED STATES with several grades under each class. The fits are:

                RC- Running or sliding fit
                LC- Locational clearance fit
                LT- Transitional clearance or interference fit
                LN- Locational interference fit
                FN- Force or shrink fit

The three fits for linear dimensions are classified as:
                FINE  MEDIUM  COARSE

The database consists of the upper and the lower limits of the tolerance
for various Nominal Sizes (up to 200 inch). The limits are in THOUSANDTH
of an inch. These tolerence limits are based on the HOLE BASIS SYSTEM.
                PRESS ANY KEY TO CONTINUE.
```

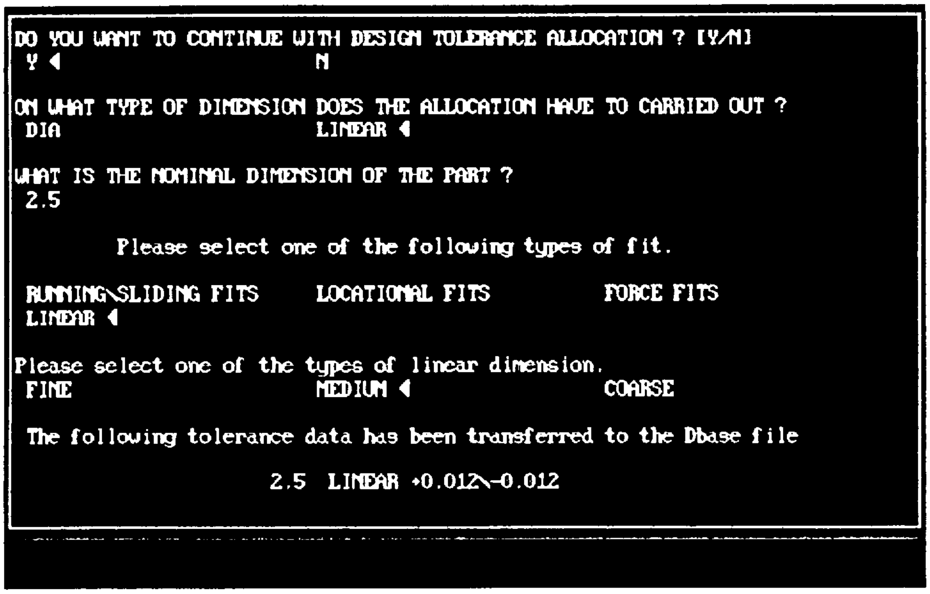
```
************************    IMPORTANT INFORMATION    ************************

The system will take the nominal size values of rotational parts from
the user input. The user will also be asked to enter the class and
the grade of fit for each basic size. Based on the user input, the
expert system will retrieve the tolerance limits and place them at the
required location in the Dbase file. The retrieved values for diametrical
dimensions will be stored as shown below:

BASIC SIZE  HOLE UPPER LIMIT\LOWER LIMIT   SHAFT UPPER LIMIT\LOWER LIMIT
            MIN CLEARANCE/INTERFERENCE   MAX CLEARANCE/INTERFERENCE

The retrieved values for linear dimensions will be stored as:

        BASIC SIZE  UPPER LIMIT/LOWER LIMIT

For any cylindrical dimension the system will retrieve limits for
both hole and shaft.

                PRESS ANY KEY TO CONTINUE.
```


C-2.

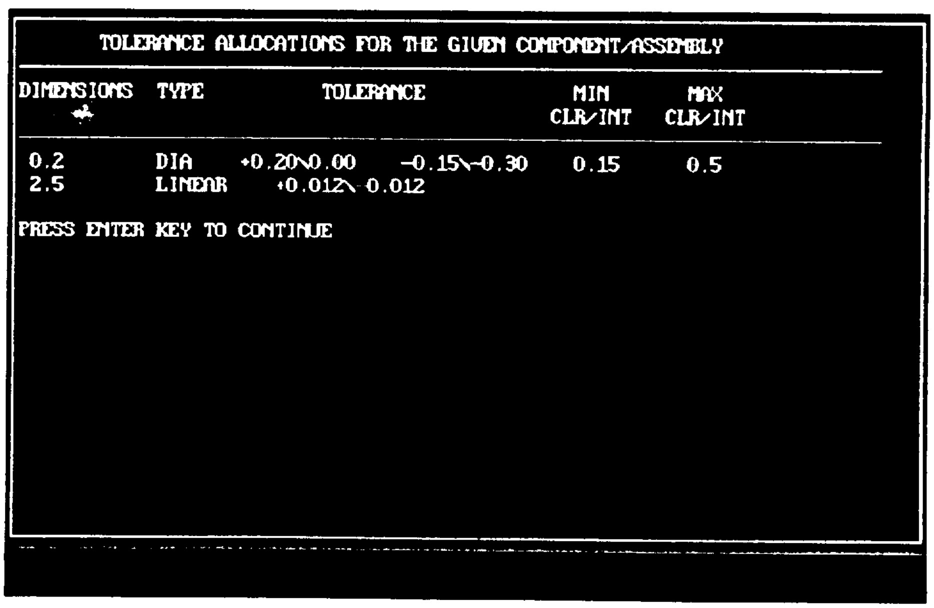
The user can continue with design tolerance allocation by selecting the 'Y' option. For tolerance allocation on radial dimensions select the option 'DIA' from the menu. For tolerance allocation on linear dimensions select the option 'LINEAR' from the menu. The following screen gives an example of design tolerance allocation on radial dimension with nominal size 0.2 inches for RC1 class of Running Fit.



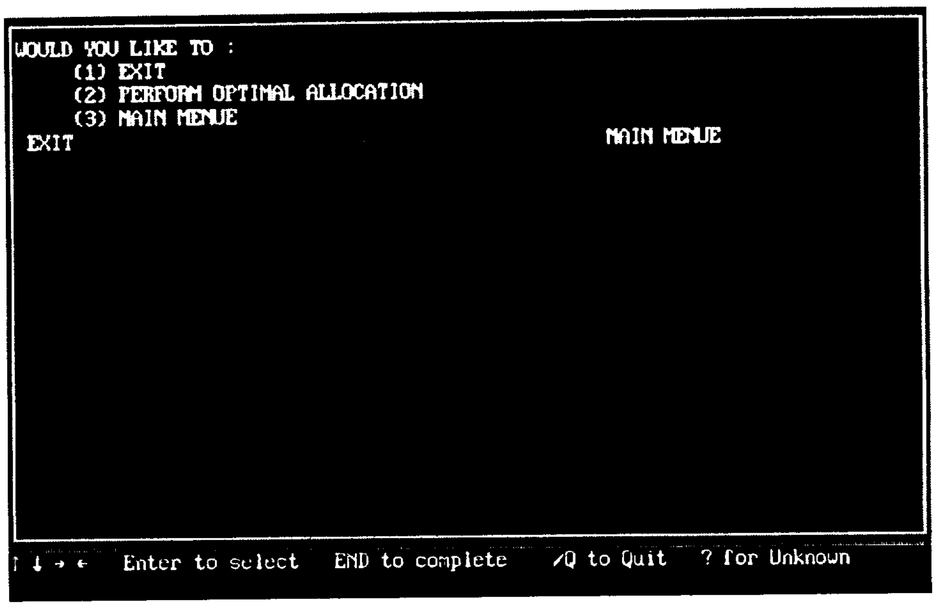
The following screen is an example of design tolerance allocation on linear dimension of nominal size 2.5 inches.



When done with design tolerance allocation, the following screen is reproduced and the user has the option to take a print-out of it from the expert system shell for further reference.



After having done with the design tolerance allocation, the user has three different options as shown in the screen below.



If the user selects to perform optimal tolerance allocation, the expert system will chain the required module (OPTIMANM.KBS) and the following screens giving the general description of this module appear.

This module allocates tolerances to each dimensions with the objective of minimizing manufacturing cost, subject to the constraint of functional requirement.

Two tolerance allocation techniques have been programmed using QuickBASIC. These methods are optimization techniques and are as follows:

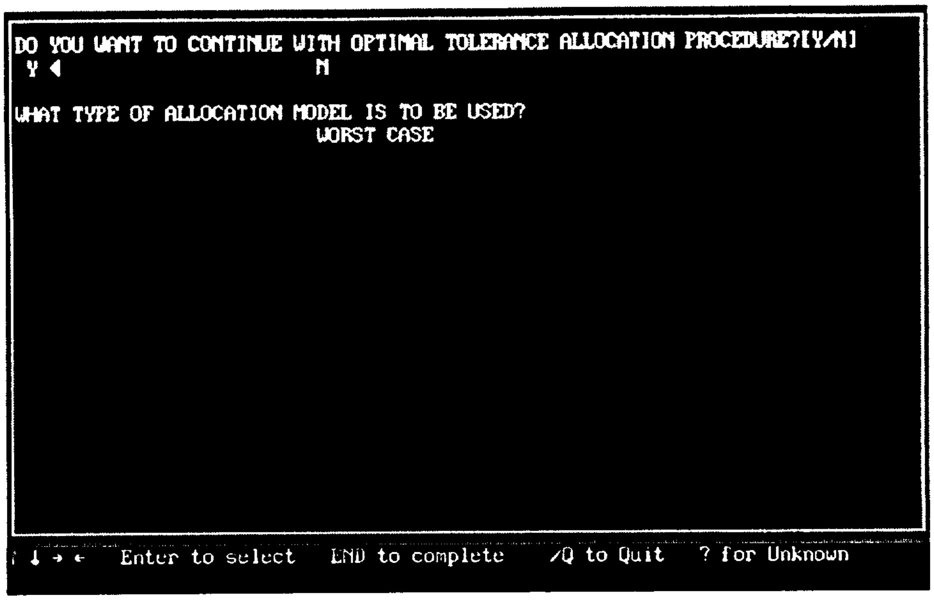
1. Linear Programming method
2. Nonlinear Programming method

Linear Programming method uses exponential cost-tolerance curve which is linearized and linear constraints based on the worst-case limits.

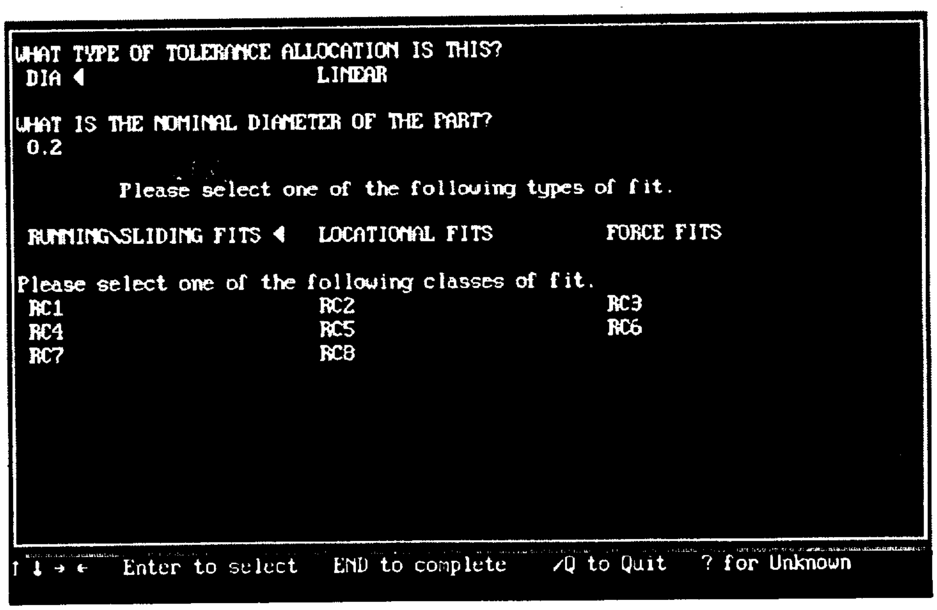
Nonlinear Programming method uses directly the exponential objective function and statistical (quadratic) constraints.

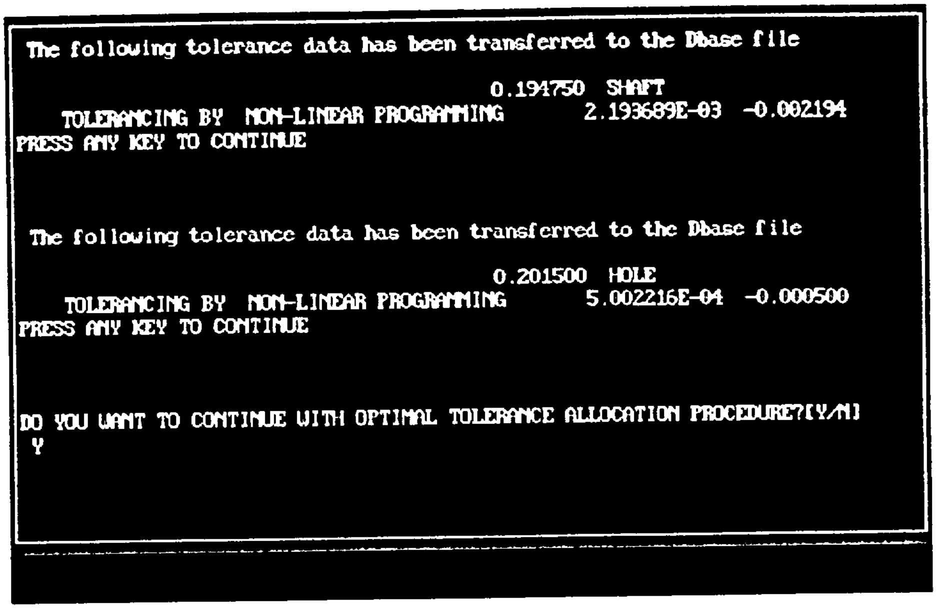
Process capability and Cost parameters are stored in the database file and they have to be tailored as per the user's available process equipment. This rule based program interfaces the data base files and the optimization programs to give optimal tolerances to the dimensions of the drawings retrieved from the database file.

As shown below, the user has the option of selecting either the Statistical (Non-linear Programming) method or the Worst-case (Linear Programming) method.

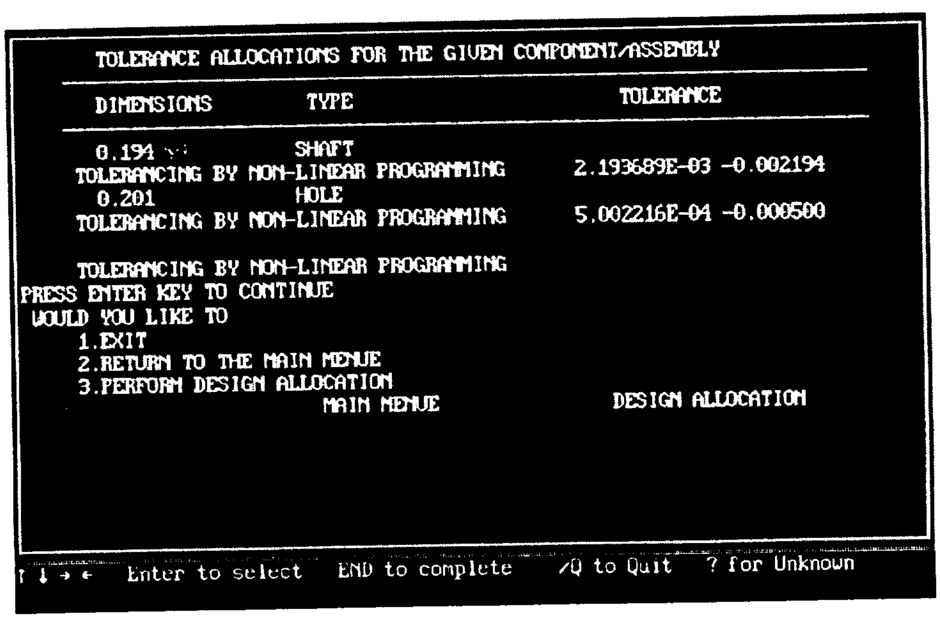


The following two screens give the user interaction

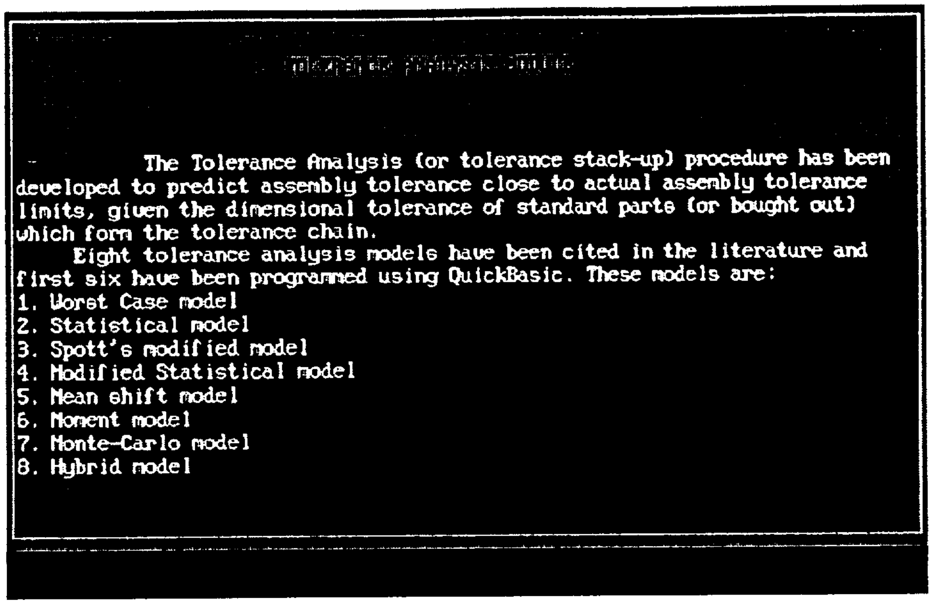




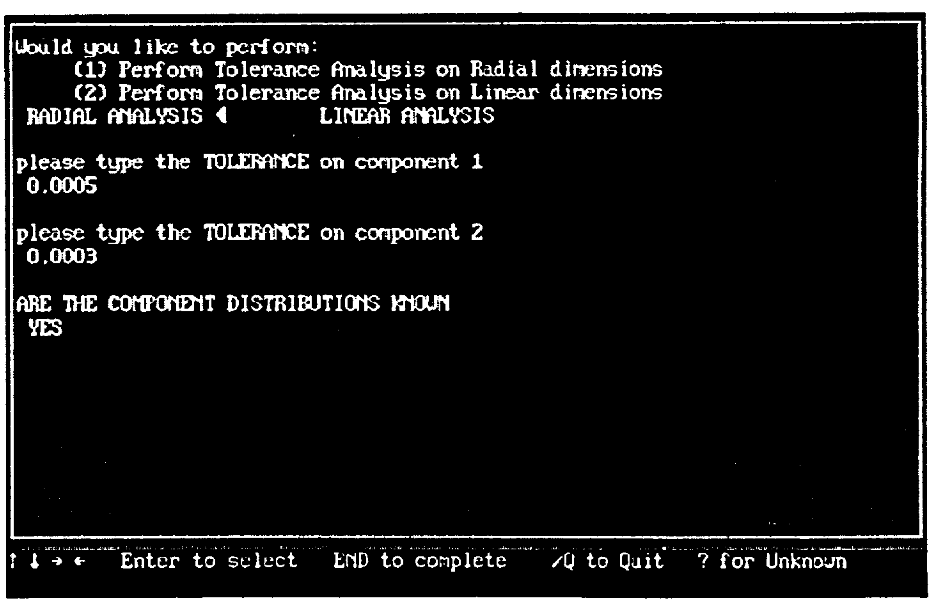
Finally the user has the option of printing the following final display table.

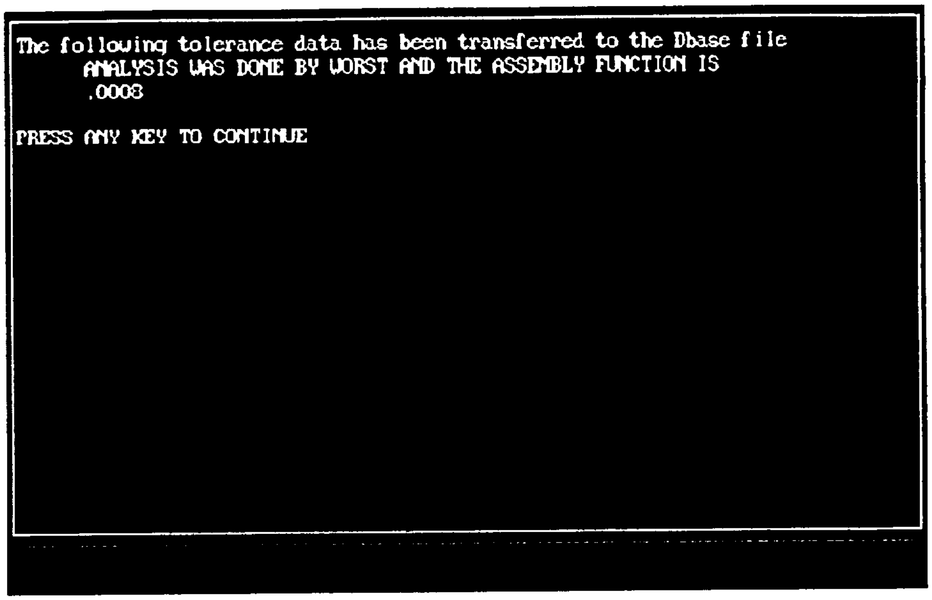


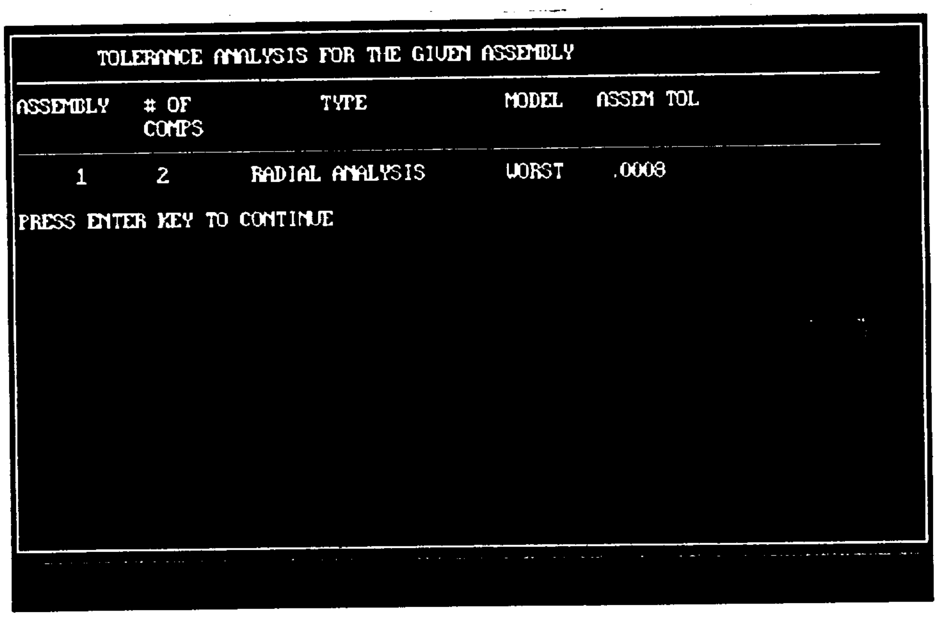
If the tolerance analysis module is run the following screen appears.



The following screens give an example of tolerance analysis on radial dimensions.




The following tolerance data has been transferred to the Dbase file
ANALYSIS WAS DONE BY WORST AND THE ASSEMBLY FUNCTION IS
.0008
PRESS ANY KEY TO CONTINUE


TOLERANCE ANALYSIS FOR THE GIVEN ASSEMBLY
ASSEMBLY
OF COMPS
TYPE
MODEL
ASSEM TOL
1
2
RADIAL ANALYSIS
WORST
.0008
PRESS ENTER KEY TO CONTINUE

5. DECISION TOOLS FOR TOLERANCE / SURFACE FINISH INTERPRETATION

5.1 Introduction

This part of the report discusses some user-friendly prototype decision tools under development to integrate procedures for tolerance/surface finish compatibility establishment, surface finish specification, process planning and machining parameter selection, NC path planning, NC program generation and graphical verification.

Tolerances and surface finish are specified on components to meet functional requirements. Generally the cost of manufacturing a component increases as the tolerance/surface finish requirements are made tighter. Keeping in mind the economy of manufacturing, it is imperative to assign tolerance/surface finish specifications prudently. Automated design of such specifications is desirable in the CAD/CAPP/CAM environment. The subsequent interpretation of these specifications and transfer to manufacturing are also extremely important in the product development cycle. This part of the report discusses the development of prototype decision aids that serve to interpret tolerances. A review of relevant literature is presented in this section.

Automatic tolerance transfer requires extensive work in tolerance representation and/or tolerance interpretation. Research in this area concentrates on the transfer of tolerance information from the design domain to the manufacturing domain; such as to insure that the manufactured dimensions fall in the range of the corresponding design dimensions (Fainguelernt et al, 1986). Such transfer must allow for two-way data flow between design and manufacturing (Truslove, 1988) . Conventional means of tolerance transfer include word of mouth, hard copy of component drawing, process specification sheet, etc. With multiple features the proper selection of the process parameters and determination of the tool path to insure transfer of design tolerance becomes complicated. Besides tolerance stacking, factors such as workpiece positioning errors, process inaccuracy errors, kinematic errors and tool wear have to be predicted and compensated for to achieve the desired specification. In practice, it is done through trial and error with large volumes of past data and expertise of an experienced process planner.

An experimental system designed by Sakal and Chow (1991) and commercial systems such as Mastercam and Smartcam are examples of CAD/CAPP/CAM systems that can generate the NC code to machine a part from the drawing. However, most such systems lack the ability to consider tolerance/surface finish requirements before preparing the NC code. The selection of processes and the generation of process plans involve considerable human intervention to insure that design tolerance and surface finish specifications are met.

The output of the tolerance allocation procedure (refer previous sections) will be eventually (automatically) integrated with the tolerance/surface finish compatibility checking routine. Accordingly, a suitable surface finish can be selected using which process parameters are selected and an automatic NC program to machine the part is prepared with little or no user interference. This is discussed in the next few sections.

5.2 Tolerance / Finish and Process Parameters

Just as tolerances, surface finish is a very important characteristic of a component that plays an important role in determining the machining processes and the process parameters. Tolerance is required mainly for functionality, whereas, surface finish is specified to meet functionality and/or improve another quality and/or satisfy appearance (Machining Data

Handbook, 1980) . The finish is defined in terms of roughness, waviness, lay and flaws. A common indicator of the roughness is the arithmetic average of roughness heights- Ra. Each manufacturing process has a characteristic surface roughness range (Machining Data Handbook, 1980). Further, a particular finish can be obtained by more than one process. Hence, selection of a process or sequence of processes to achieve a desired finish depends on several factors such as cost, machinery/tool material availability, manufacturing time etc. In general, the cost of producing a fine finish is higher than the cost required to produce a coarser finish.

5.2.1 Relation Between Cost and Finish

The surface finish / cost relationship is consistent with the behavior of tolerances with respect to cost and it is possible to relate finish with tolerances. Moreover, it may not be feasible to hold a tolerance of 0.0001 inch on a part which is machined to an average roughness of 125 microinch rms. Hence, a requirement for the accurate measurement of a dimension is that the variations introduced by the surface roughness should not exceed the tolerance placed on that dimension. If this is not the case, the measurement of the dimension will be subject to an uncertainty greater than the required tolerance. To maintain consistency between the variables a table is created based on a plot (between and surface finish and tolerance) developed by Trucks (1974).

It is thus important to verify on surfaces that have a tolerance specification, that the surface finish specification is compatible with the tolerance specification. If this check is not built in as an integral part of the system, chances that any incompatibility might go unnoticed until a later stage are high. Any downstream correction of specifications is not only expensive, but it might upset the entire production plan.

The production of components adhering to certain tolerance specifications seems to be a routine task; however, no direct relationship can be derived between tolerances and cutting parameters. Utilizing the relationship between surface finish and the process parameters and the consistency between tolerance and finish an indirect relationship is established in this research between the parameters and tolerances. Hence, an indirect tolerance interpretation and corresponding transfer is achieved.

5.3 Finish Interpretation and Transfer

An overview of the system for finish interpretation and transfer and its proposed relationship to tolerance design is shown in figure 1. The integrated system has four modules: a CAD module, a DATABASE module, an EXPERT module and the Program module. The user interacts with each of these and they in turn, interact with each other. Since, one of the main objectives is to study the feasibility of integrating the various modules to achieve effective tolerance interpretation the scope of the system was limited to symmetric rotational parts with steps (no tapers). Further, only dimensional tolerances for radial features are considered. The operations are limited to facing, rough, semi-finish and finish turning operations for external surfaces and centering, drilling, rough, semi-finish and finish boring operations for internal surfaces. Another assumption is made that the bar stock is continuously being fed.

The 2-d drawing of the component drawn in AutoCAD (1989) is used as one of the inputs to the system. Another (desired) input is an allocated tolerance file from the tolerance design procedure. At present the proper interpretation of the output from the allocation procedure is under investigation. This is necessary before the tolerance design and finish interpretation procedures are properly integrated.

A program written in AutoLISP does feature definition, by writing the vertex points of the external and internal features of the upper half of the component into certain files. The database houses the various databases and consultation mechanisms for surface finish verification and validation. The databases contain the machining parameters for rough, semi-finish and finish turning, drilling, rough, semi-finish and finish boring. These databases are taken from the Machining Data Handbook (1984), for various combinations of work material, tool material and work material hardness. These are conservative estimates and may not be universally true.

The system was developed to:

(1) Verify the tolerance/surface finish compatibility and if not compatible, recommend a surface finish value that is compatible with the dimensional tolerance. Next, the process plan based on the revised component specifications is generated and the cutting parameters, based on surface finish requirements are selected and recommended.

To verify the tolerance/finish compatibility a program searches a database to determine the range of surface finish (R_a) that would be most suitable for the specified tolerance to be maintained and measured. The user is prompted to input a value for Ra within the recommended range; if there is no surface finish requirement, then it is defaulted to the highest value within that range. This is logical, to insure reduced costs.

An expert module houses the rules for selecting the operations that would result in the specified surface finish. This also provides an user interface for selecting work materials, tool materials, work material hardness and for modifying and validating the cutting parameters selected. A knowledge based program contains the knowledge to determine the processes needed for each external feature and for retrieving and modifying the cutting parameters for each process selected. Simple rules are used to determine the processes. The values of speed, feed and depth of cut matching the conditions recommended are retrieved and displayed. The user has the choice of accepting these or changing any of them. The number of operations, the speed, feed and depth of cut for each operation are written into a separate file. If internal surfaces exist, this is identified and a similar consultation is done for each internal surface and the recommended values are written into a corresponding file. More details are provided in Kumar (1993).

(2) Generate the cutter path information necessary to machine the component, by using the corresponding machining parameters for each surface and to graphically simulate the cutter path to verify the cutter path information.

Using the cutting parameters to machine each segment, a program module has the necessary logic to determine the cutter path information for the different (facing, turning, boring and drilling) operations.

The **facing** routine generates cutter path for facing (0.2" depth of cut). The faced edge can be used as a reference datum for subsequent measurements. The **centering** routine is activated by the length/diameter ratio of the component. The **completurn** (complete turn) routine is used for turning the excess stock over the maximum diameter. High metal removal rate is used to cut down on production time. A **drilling** program drills a through hole around the axis of the component, a prerequisite for boring. A **regturn** (regular turn) routine writes the cutter path information for external turning. For each segment of machining, it uses the corresponding recommended machining parameters from a previously created file. An **inside** routine uses similar procedures for internal operations.

The logic for determining the cutter path for external machining and the sequence of motion of the cutter to machine a component are discussed in detail in Kumar (1993). The determination of the check surface for each operation is also discussed in that paper. After each surface is machined, a subroutine called "Updtsttus" (Update status) is called to update the values of the cutter direction, speed, feed and the surface being machined. An additional line of code to move the cutter from the last point of the current cycle to the starting point of the next cycle is automatically inserted.

Different programs using similar logic are utilized to generate the cutter path information for boring, drilling and facing. In the present prototype, only the following words of a standard NC word code are used: N, G, X, Z, S, F. It is to be noted that an actual machine downloadable NC code is not created but a sample for demonstration purposes. However, depending on the particular machine, the actual code may also be prepared without much difficulty. The logic for determination of the cutter path is modular,and hence, flexible for future modifications.

The graphical simulator is designed to verify the cutter path and highlight any portion of the roughstock left unmachined, any unnecessary machining, or any tool-work material collision. The different depths of cut and feed rates corresponding to each machining operation are also clearly demonstrated.

6. PROCESS SELECTION WITH TOLERANCING

6.1 Introduction

Systematic methodologies are needed for modeling processes to evaluate processing alternatives for given tolerances. How to provide this information to obtain more robust and cost-effective designs is one of the most widely recognized needs central to the concept of design for manufacturability (Zhang and Wang (1993)). In accordance with this concept, we present a methodology for allocation of tolerances and simultaneous selection of manufacturing processes for minimum cost. In this way the manufacturing concerns are embedded in the design process itself.

The tolerance specification on individual components have a significant impact on the cost of the assembly. The specified tolerances govern the selection of manufacturing process for that component. Several processes are usually available for manufacturing a particular component with some desired feature. The problem to be addressed is selection of the best possible combination of processes for the different components of the assembly and allocation of tolerances on these components so that the total cost is minimized and the required assembly performance is met.

As an example, consider the configuration of an assembly as shown in Figure 6.1. The assembly is made up of three components, each of which can be manufactured using three different processes as shown in Figure 6.2. The tolerance allocated on each component should be such that the final assembly tolerance should not exceed certain value, say t_a (for the worst case stack-up). Among the several combinations of processes (in this case 27 combinations) available, we need to select a best combination and allocate tolerances on each component so that the cost is minimized and the required assembly tolerance (t_a) is not exceeded. It is therefore a discrete combinatorial optimization problem.

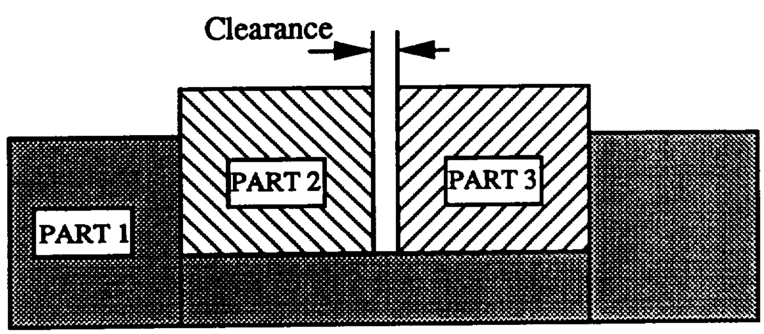


Figure 6.1 Sample assembly of three parts

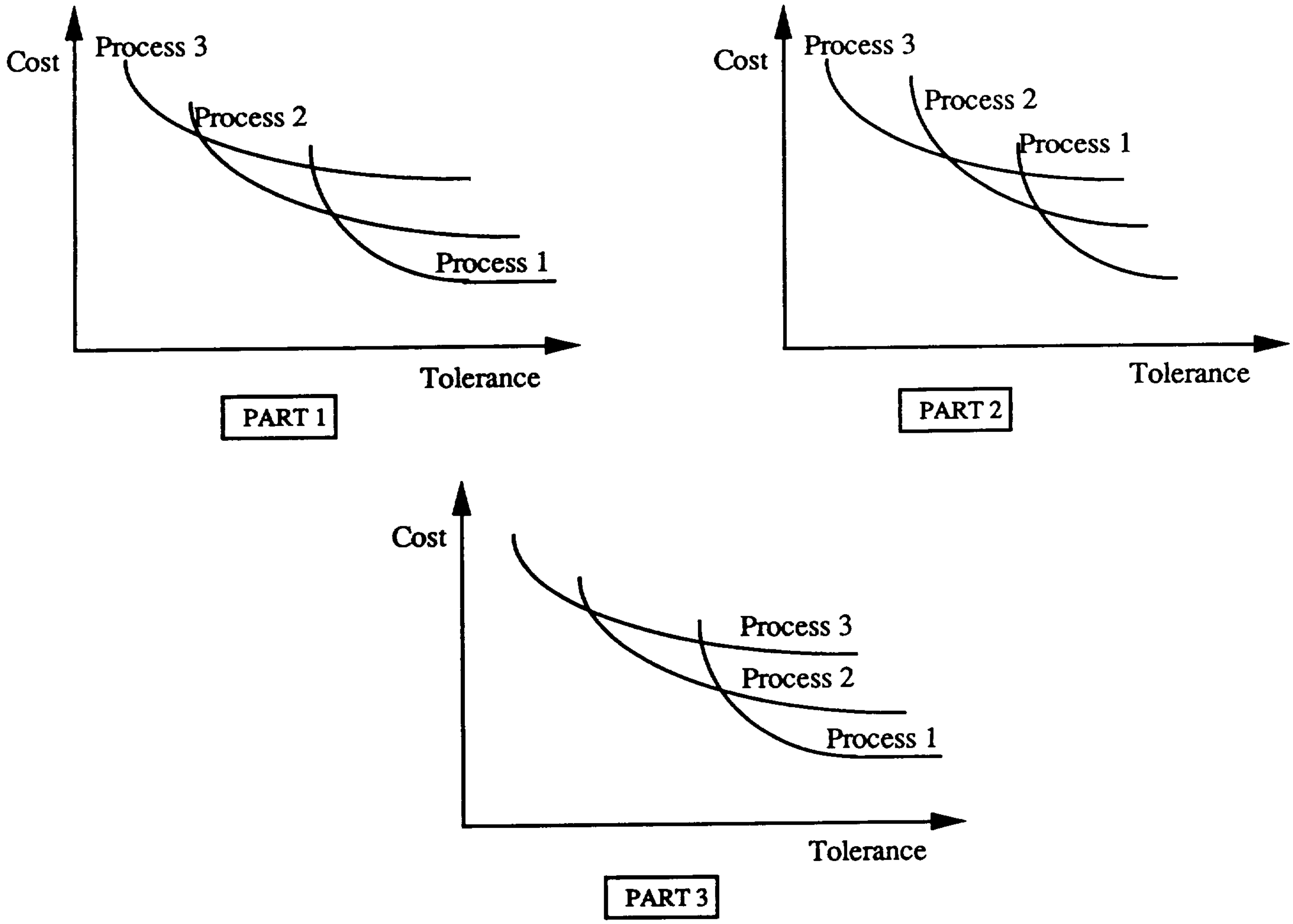


Figure 6.2 Cost-tolerance curves of alternative processes to manufacture the three different parts of the assembly shown in Figure 6.1

6.2 Model Development

In this study, an exponential cost versus tolerance relationship was used. This relationship is expressed as:

$$C = ae^{(-bt)}$$

where a,b are process specific constants

and t is the tolerance

The tolerance-specification model is given as

$$Min \sum_{i=1}^{n} \sum_{j=1}^{p(i)} x_{ij} a_{ij} e^{-b_{ij} t_{ij}}$$

subject to

$$\sum_{i=1}^{n} \sum_{j=1}^{p(i)} x_{ij} t_{ij}^2 \leq t_a^2 \ldots\ldots\ldots\ldots\ldots\ldots (statistical)$$

or

$$\sum_{i=1}^{n} \sum_{j=1}^{p(i)} x_{ij} t_{ij} \leq t_a \ldots\ldots\ldots\ldots\ldots\ldots (worstcase)$$

$$tmin_{ij} \leq t_{ij} \leq tmax_{ij}$$

$$\sum_{j=1}^{p} x_{ij} = 1 \qquad\qquad \forall\ i = 1,2,\ldots,n$$

$$t_{ij} \geq 0$$

$$x_{ij} = 1 \quad \textit{if process } j \textit{ is used for manufacturing comp } i$$
$$= 0 \quad \textit{otherwise}$$

where

n number of parts in the assembly

p(i) number of processes for manufacturing part i

a_{ij},b_{ij} parameters in the cost-tolerance function associated with process j to manufacture part i

x_{ij} decision variable that selects the j^{th} process from those available to manufacture part i

t_{ij} tolerance for i^{th} part and j^{th} process

$tmin_{ij}$ lower tolerance limit for process j on part i

$tmax_{ij}$ upper tolerance limit for process j on part i

t_a overall assembly tolerance

6.3 Slope-Based Methodology

In this section we develop a new algorithm, called the slope-based algorithm, for solving the above discrete optimization problem. This algorithm is computationally very efficient and is useful for solving the simultaneous process selection and tolerance optimization problem in practice.

The main premise in using this algorithm is that higher the tolerance lower the manufacturing cost. Here we start by allocating the maximum tolerances (very high tolerances) on each of the components in the assembly. Even though this would yield an infeasible solution as far as the assembly constraint is concerned, we systematically reduce this infeasibility by reducing the tolerance on specific components in small steps until the assembly constraint (could be either statistical of worst case constraint) is satisfied. The component on which the tolerance is to be reduced is so selected such that the increase in the total cost is minimum for that particular step reduction of the tolerance. The solution obtained at the final iteration when feasibility is achieved is taken as the best solution.

First it is necessary to define the cost model for a combined process for each component. As explained by Bjorke (1989), a cost model for a combined process means a plot of the manufacturing cost as a function of machining precision (tolerance) under the assumption that different processes yield different precision. For any given tolerance, there is only one manufacturing process that yields the lowest possible cost. A cost model for the combined process is given in Figure 6.3. This model consists of discontinuities. The points of discontinuities define the tolerances at which two processes are equivalent as far as cost is concerned. The intervals between the break points define the economical ranges of individual processes. In Figure 6.3, process 1 should be used when the required tolerance is greater than T_{12}, process 2 should be used for a tolerance in the range between T_{12} and T_{23}, and process 3 should be used for a tolerance narrower than T_{23}. Thus as shown in Figure 6.4, a single continuous cost versus tolerance function for all the different manufacturing processes for a single component is obtained. Similar type of curves exist for all other components in the assembly. These cost-tolerance functions are neither concave nor convex.

After having obtained the above mentioned cost-tolerance function for each component, these cost functions are approximated by a sequence of linear function as indicated in Figure 6.5. To obtain higher precision, the linear function should be as close as possible to the original cost-tolerance curve. This can be achieved by linear approximation of the cost-tolerance curve over smaller intervals. Next the slopes of each of the linear segments of the cost-tolerance curves for each component are calculated and the range of tolerance over which each linear function is defined is noted. The algorithm in pseudo-code is given in Figure 6.6.

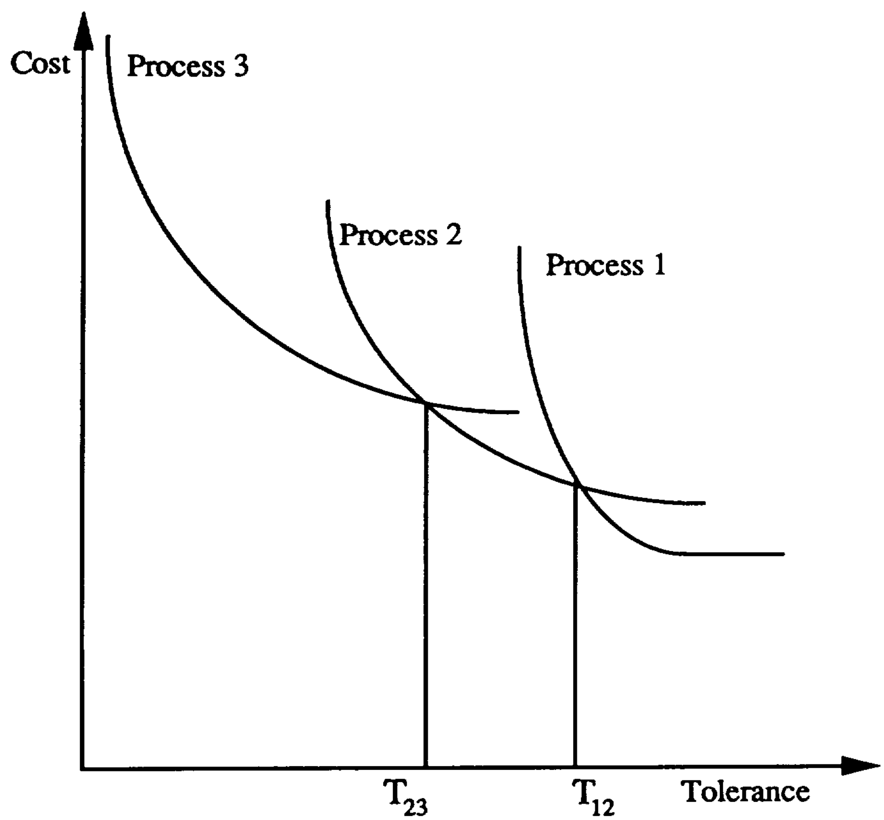


Figure 6.3 Cost model for a combined process (after Bjorke, 1989)

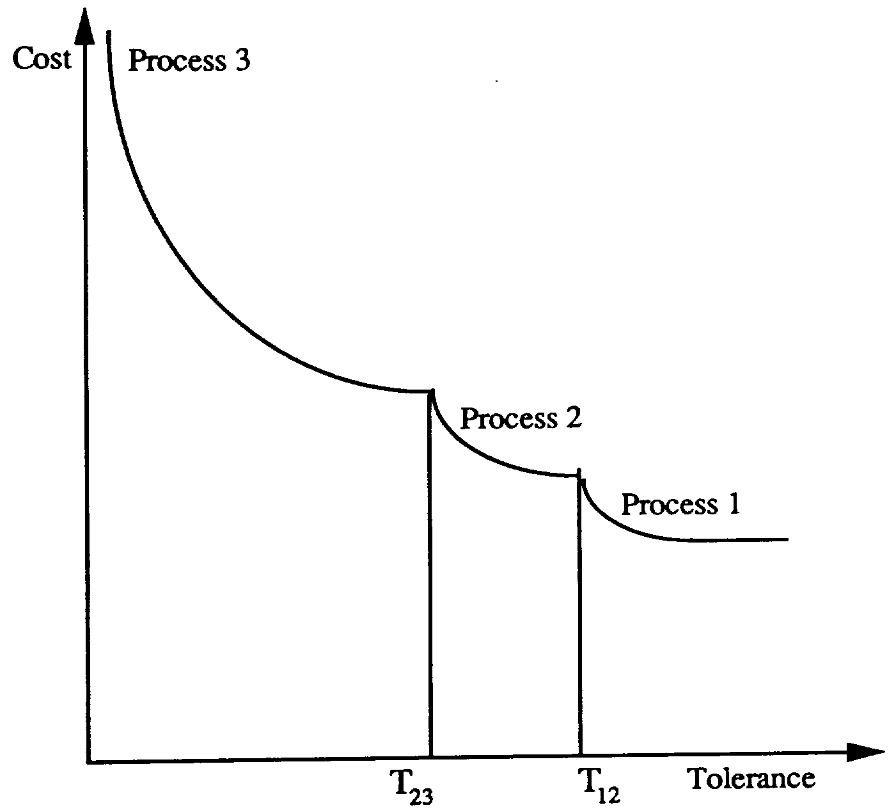


Figure 6.4 Continuous cost vs tolerance curve derived from Figure 6.3

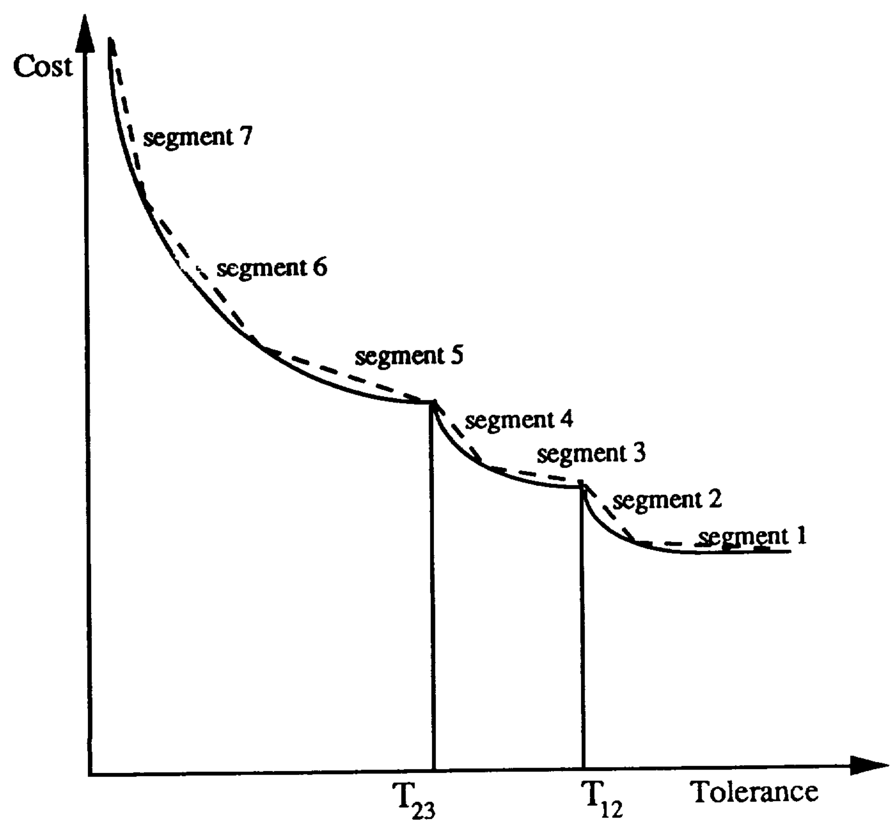


Figure 6.5 Sequence of linear approximations of the cost-tolerance curve of Figure 6.4

SLOPE-BASED ALGORITHM

1. Begin by assigning maximum (very high) tolerances (t_i) on each component using the minimum cost segment (i.e. segment #1);

2. Do the following while infeasibility exists (the solution is infeasible if $\Sigma t_i^2 > t_a^2$ in case of statistical type of constraint or if $\Sigma t_i > t_a$ in case of worst-case type of constraint, and the infeasibility is $\alpha = \Sigma t_i^2 - t_a^2$ for statistical type of constraint or $\alpha = \Sigma t_i - t_a$ for worst-case constraint)

   2.1 From among the cost-tolerance segments for each component, find the segment which gives the least increase in cost per unit decrease in tolerance;

   2.2 For the segment selected in 2.1 calculate the increase in cost that would result if the tolerance on this component were to be reduced by an amount equal to the range Δ over which that segment is defined. The infeasibility would decrease by amount $\delta = (t_i - \Delta)^2$ for statistical constraint or by an amount $\delta = \Delta$ for worst-case constraint; if $\delta > \alpha$ then calculate the cost increase for the corresponding reduction in tolerance that would result in reducing the infeasibility by amount $\delta = \alpha$;

   2.3 For each of the remaining components calculate the corresponding increase in cost that would result if the tolerance on each of these components were to be reduced by an amount that would lead to the same reduction in infeasibility (amount δ);

   2.4 For the component that leads to the least increase in cost, decrease the tolerance by the amount determined in either of steps 2.2 or 2.3 and select the corresponding cost-tolerance segment;

3. Now even though the feasibility is achieved, from the present cost-tolerance segments for each component, select the one which would give the least increase in cost per unit decrease in tolerance. For the corresponding component, calculate the increase in cost that would result if the tolerance were to be further reduced by an amount that would lead to another cost-tolerance segment. Also for each of the remaining components calculate the decrease in cost that would result if the tolerance on them were to be increased such that an exact feasible solution were to be obtained. If the decrease in cost is more than the increase, corresponding increase and decrease in tolerance on the selected component should be carried out. This process is repeated until there is no cost saving resulting from the above moves.

Figure 6.6. Slope-based algorithm in pseudo-code

The algorithm as described above, is found to be computationally very efficient. The computation time is directly dependent on how far the solution obtained in the first step is away from the feasible region. However, this algorithm has certain limitations:

1. It can not treat combined cost-tolerance functions with gaps as shown in Figure 6.7. This case arises in case of nonoverlapping cost curves for individual components.

2. It needs to be modified for assemblies with interrelated tolerance chains, that is, assemblies which are described by more than one assembly function with shared dimensions.

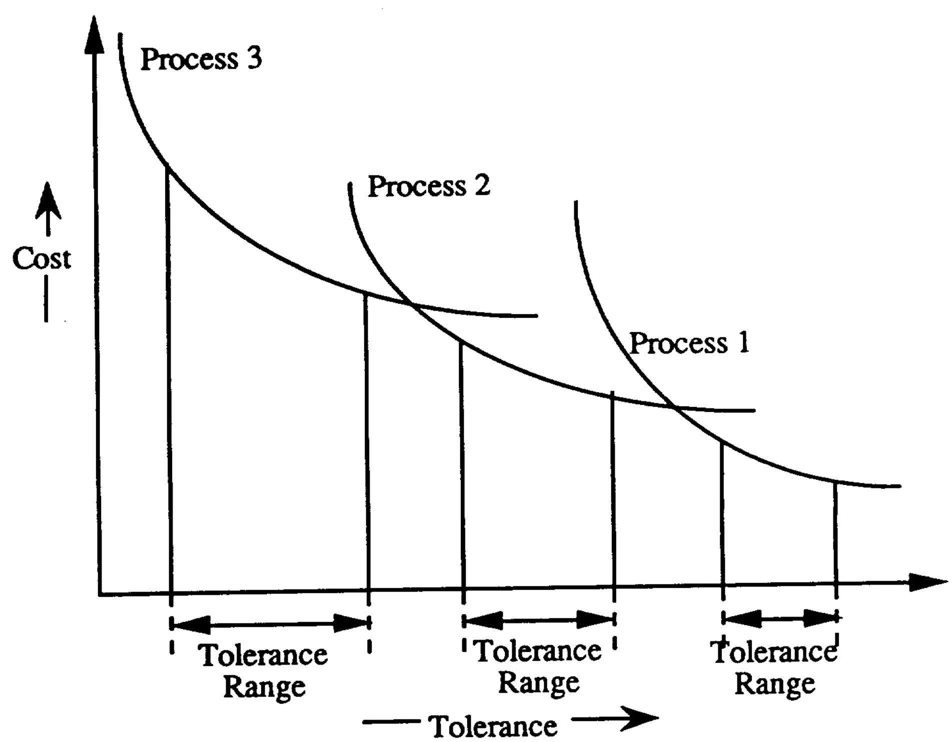


Figure 6.7 Bounded process cost curves

6.4 An Example

An example is solved using the slope-based algorithm for simple tolerance chains. In this example we have used the statistical type of constraint, namely,

$$\sum_{i=1}^{N} t_i^2 \le t_a^2$$

where N is the total number of components in the assembly

t_i is the tolerance on component i

and t_a is the assembly tolerance.

Exponential type of cost-tolerance function of the form

$$a_i e^{(-b_i t_i)}$$

is assumed.

Cagan and Kurfess (1992) have solved a friction wheel example using the simulated annealing technique they proposed. In this example there are four components forming an assembly. Each of these components can be manufactured by three different processes and each of these processes have a reciprocal cost-tolerance relationship. Also a worst case stack-up of tolerances is assumed. We solved this problem using the slope-based algorithm. Figure 6.8 shows the combined cost-tolerance functions and Figure 6.9 shows the linear approximations of these functions as used in the slope-based algorithm. Figures 6.10a through 6.10c give the results obtained using the slope-based algorithm.

The computational time to solve this example ranged from only a fraction of a second to a maximum of two seconds.

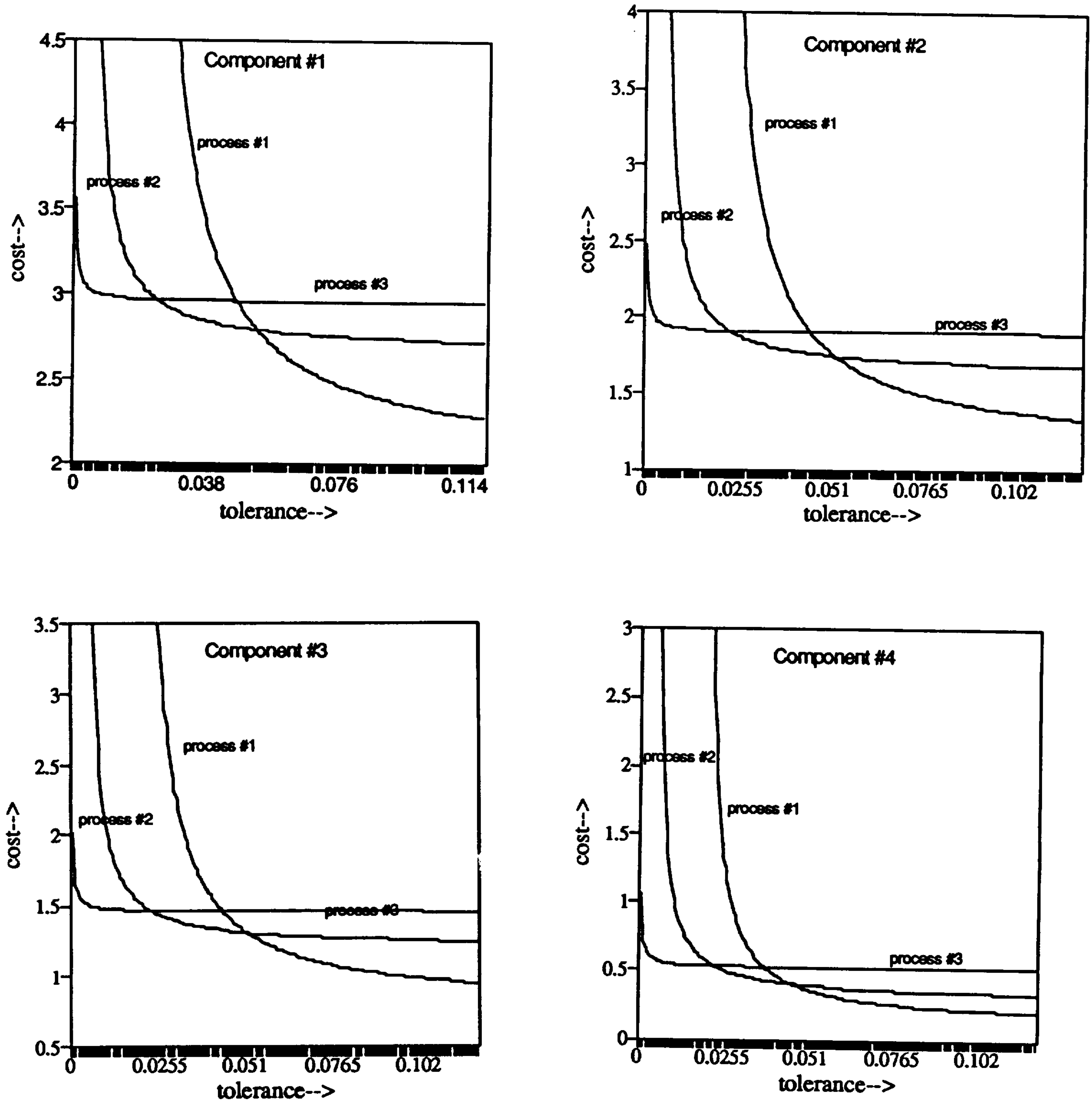


Figure 6.8 Cost-tolerance function for different components in Example 1

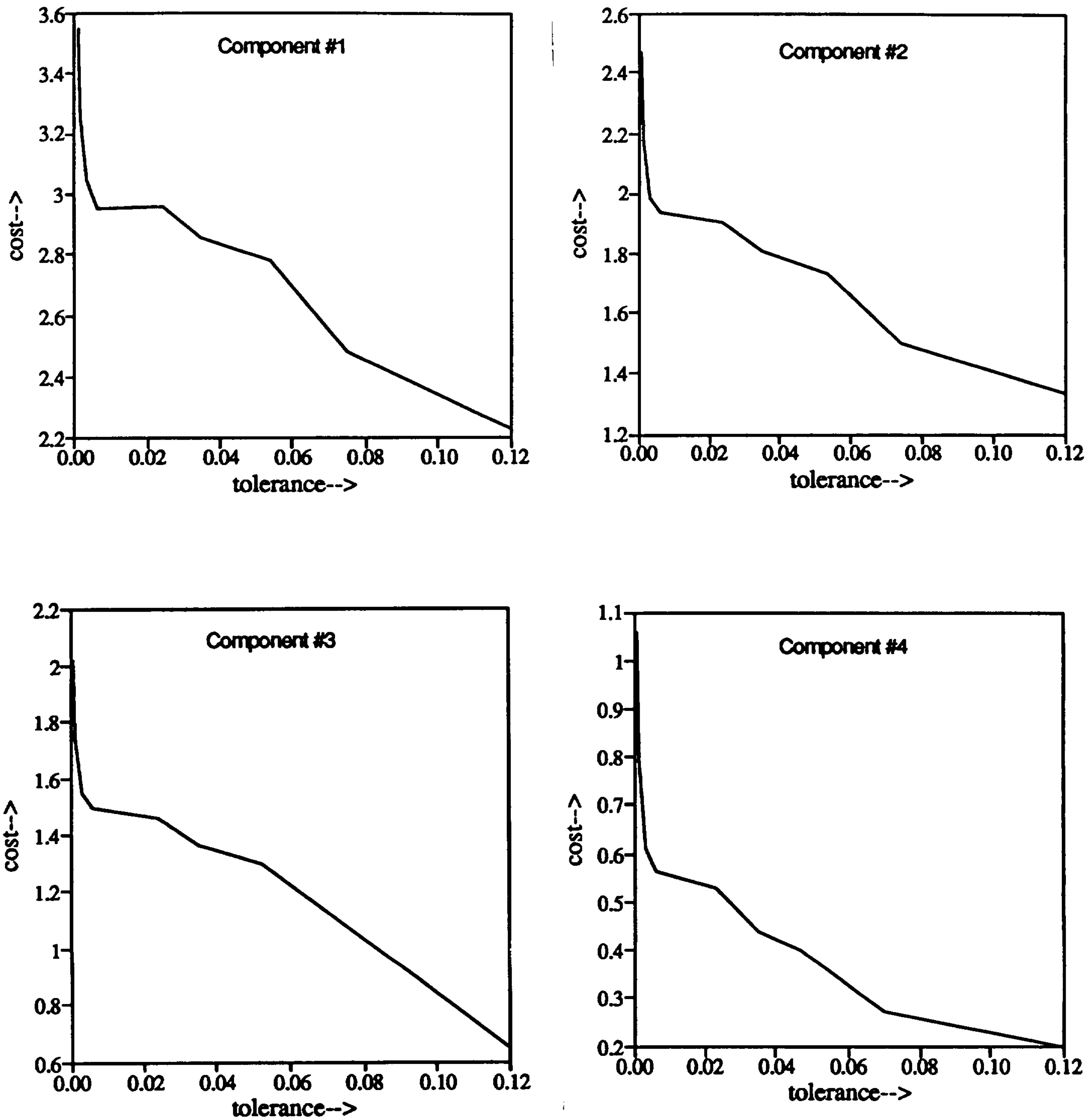


Figure 6.9 Linear approxiamtion of the cost-tolerance functions in Figure 6.8

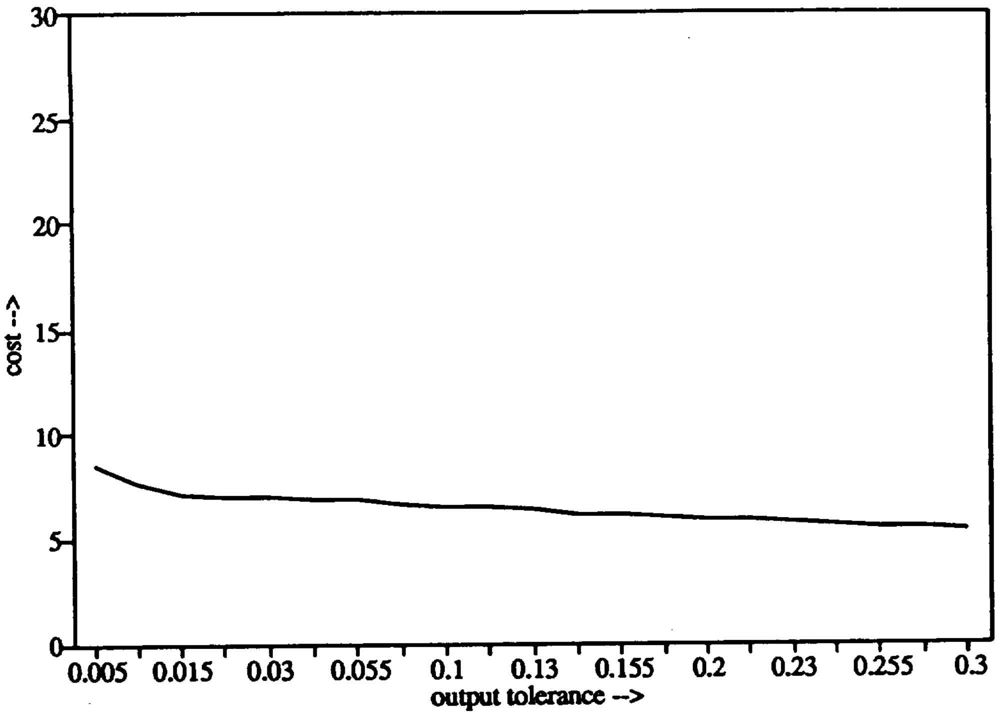


Figure 6.10a Optimal cost versus tolerance using slope-based method

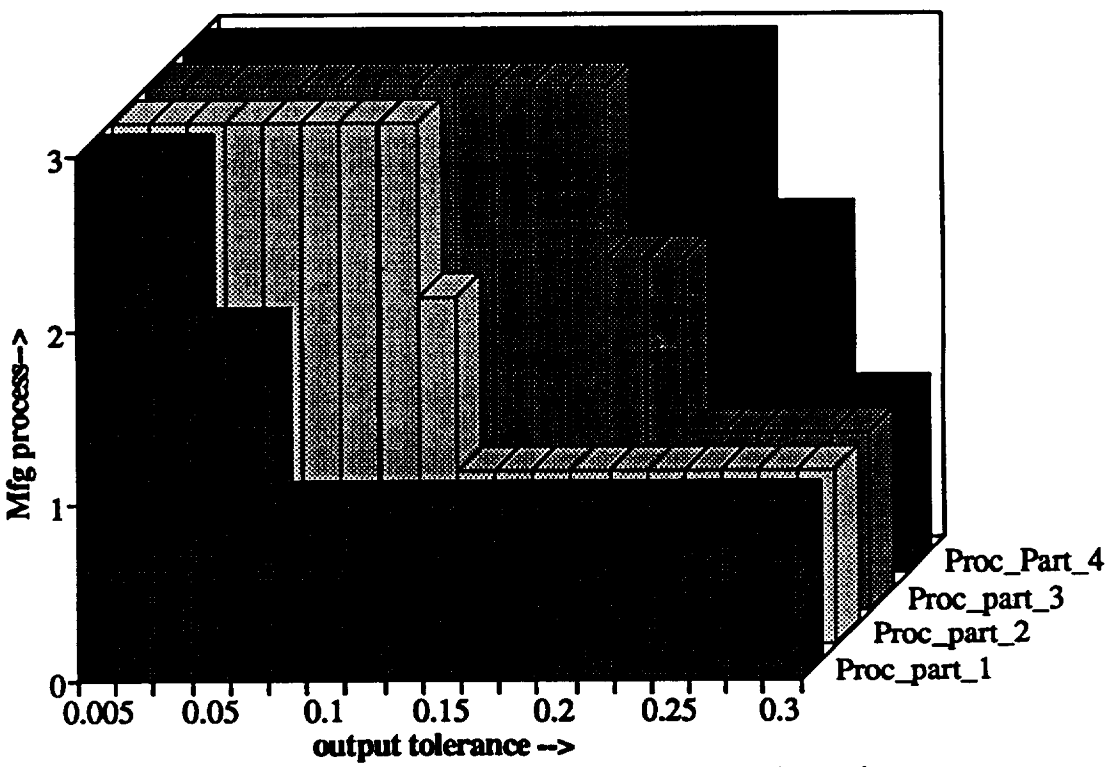


Figure 6.10b Optimal process vs output tolerance for each component using slope-based method

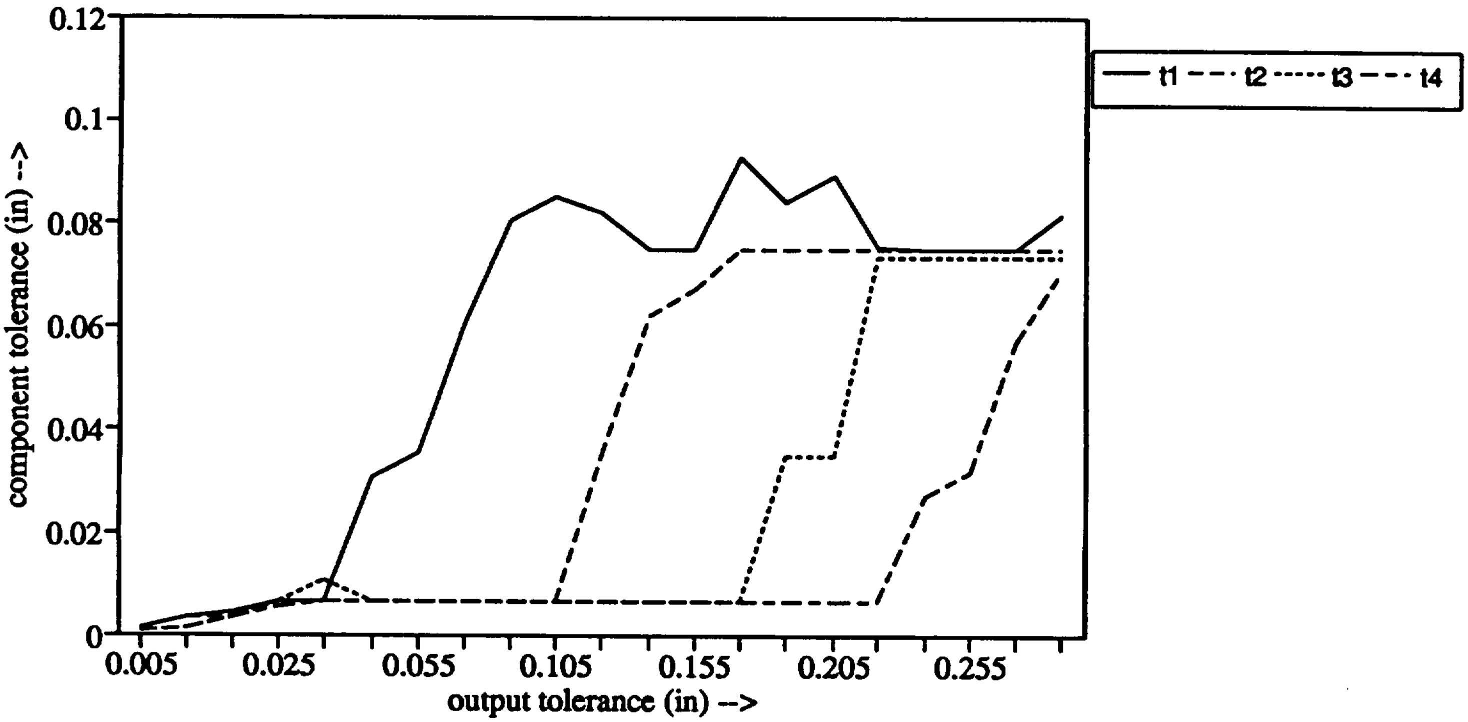


Figure 6.10c Optimal allocated tolerance vs output tolerance for each component using slope-based method

6.5 Interrelated Tolerance Chains

So far we have only considered simple tolerance chains. However, assemblies with interrelated tolerance chains are common. These types of assemblies have more than one assembly function, i.e. there are more than one assembly constraint, with shared dimensions (components). The wheel mounting in Figure 6.11 is an example of a design with interrelated tolerance chains (Bjorke, (1989)). The corresponding tolerance graph is shown to the right in the figure.

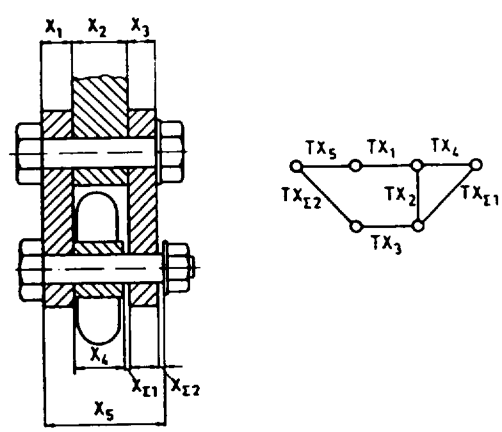


Figure 6.11. Example of design with interrelated tolerance chains (from Bjorke (1989))

In this design there are two assembly functions ($TX_{\Sigma 1}$ and $TX_{\Sigma 2}$) that must be satisfied as per the worst case or the statistical model with the link TX_2 shared by both the loops. Therefore, mathematically the following constraints must be satisfied:

$$TX_1 + TX_2 + TX_3 + TX_5 \le TX_{\Sigma 1}$$
$$TX_2 + TX_4 \le TX_{\Sigma 2} \qquad ....(worst\ case)$$

$$TX_1^2 + TX_2^2 + TX_3^2 + TX_5^2 \le TX_{\Sigma 1}^2$$
$$TX_2^2 + TX_4^2 \le TX_{\Sigma 2}^2 \qquad ....(statistical)$$

6.6 Slope-Based Methodology for Interrelated Tolerance Chains

The cost-tolerance relationship is linearized for all the parts as explained previously. The following steps are then performed to get the best solution.

1. Initially allocate maximum tolerances on each dimensions in all the loops.
2. Consider the loops which have the maximum number of shared dimensions (components) and for the shared dimensions in these loops allocate tolerances such that for given reduction in infeasibility the increase in the manufacturing cost is minimum (using slope-based method described earlier). It is to be noted that the infeasibility in all the loops with the common dimension under consideration is decreased if tolerance on this common dimension is decreased while the cost only counts once.
3. After having allocated tolerances on the common dimensions, consider each loop separately and allocate tolerances on the remaining dimensions in the loop using the slope-based method and with the restriction that the tolerances on the common dimensions cannot be increased from what is already allocated on them.

6.7 An Example

This methodology was applied to solve a tolerance allocation example of interrelated chains with two loops. A similar example is solved by Zhang and Wang (1993) using simulated annealing technique.

In this example there are a total of five dimensions (components) with three dimensions common to both the loops. The tolerance graph for this example is shown in Figure 6.12. Components 1, 2 and 5 can be manufactured by three different processes whereas components 3 and 4 can be manufactured using two different processes. The continuous cost-tolerance relationship for each of the components is shown in Figure 6.13. The worst-case model is used so that the assembly constraints would be represented by the following equations:

$$t_1 + t_2 + t_3 + t_4 + t_5 <= 0.025$$

$$t_3 + t_4 + t_5 <= 0.015$$

The solutions obtained for this example are shown in Table 6.1.

Table 6.1. Solutions for example on interrelated chains

Comp	Tolerance	Process
1	0.006	3
2	0.007	2
3	0.002	1
4	0.003	1
5	0.007	3
	total cost = 14.81	

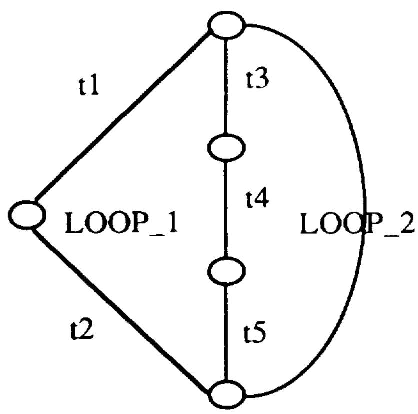


Figure 6.12. Tolerance graph for interrelated tolerance chain of example 2

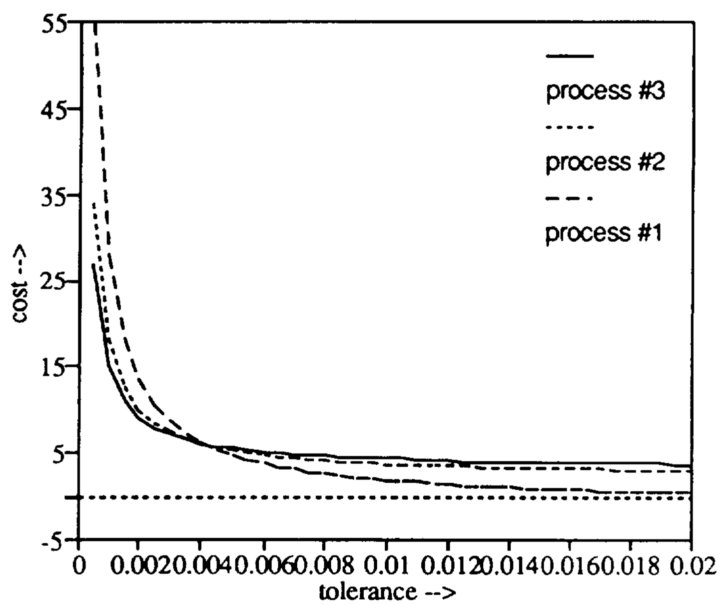


Figure 6.13 Cost-tolerance function of components in Example #2

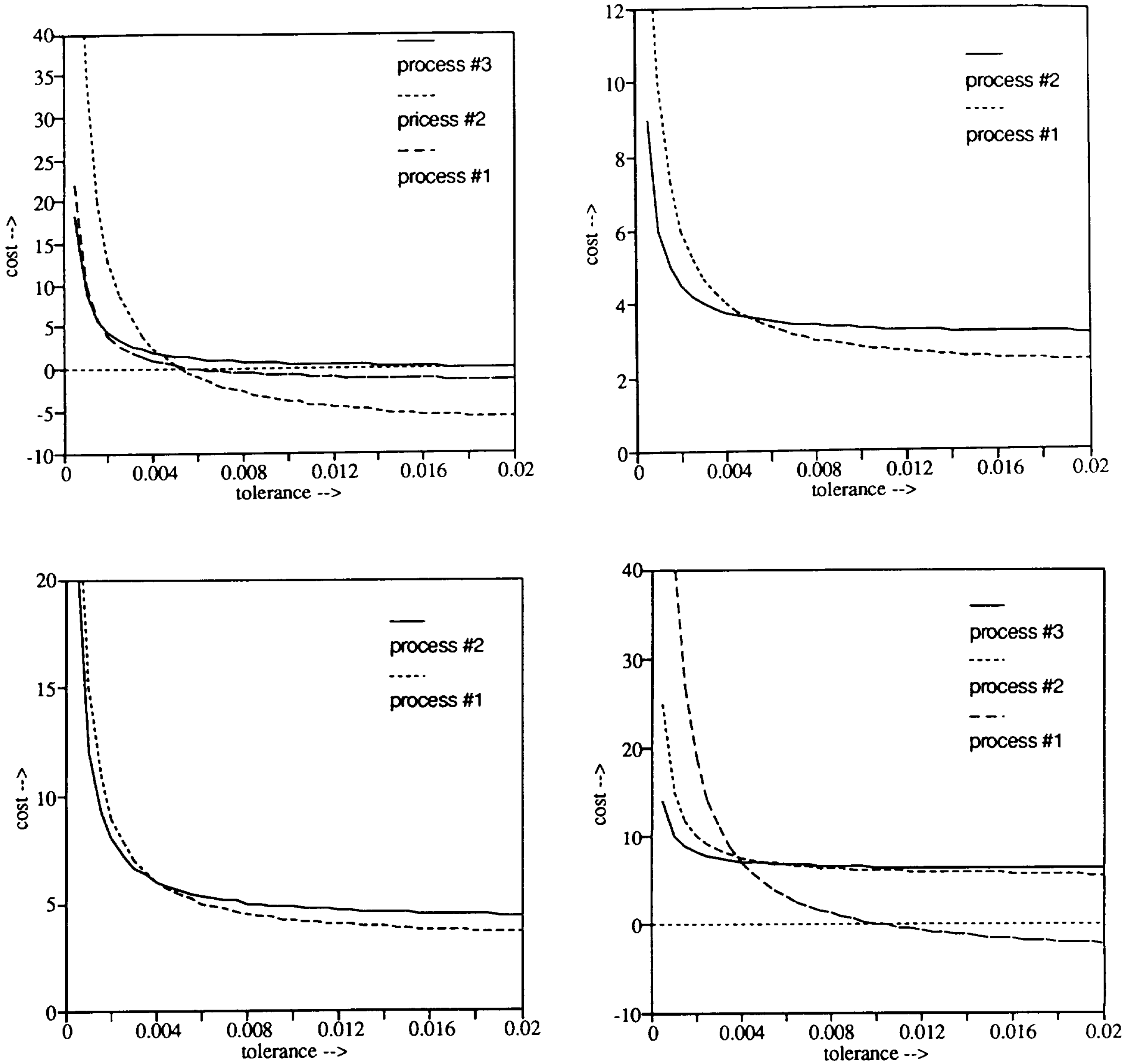


Figure 6.13(cont) Cost-tolerance functions of components in Example #2

7. CONCLUSIONS AND RECOMMENDATIONS

1. Specification of dimensional tolerances for manufacturing parts is of vital importance to discrete part manufacturing industries.

2. The principal purpose of this project was to develop a prototype for integrating the various sources of knowledge to automate tolerance specification. An user-friendly prototype software is developed to assist design and manufacturing personnel in tolerance allocation and analysis. Using the software, tolerance allocation to dimensions of components can be performed based on standard handbook data or based on optimization models. The tolerance analysis procedure uses an interactive decision support procedure to aggregate component tolerances and determine assembly tolerances. All the prototype software are developed within a Zenith 386SX computer environment using easily accessible and standard software. At present, the system is limited to size tolerance specification, linear and radial fits and concentric rotational part types. In the future, design issues for form (flatness, roundness, etc.) and geometric tolerances (concentricity, position, perpendicularity) must be investigated covering a wide range of part types. Integration with CAD tools must also be studied.

3. A new methodology for tolerance allocation and simultaneous process selection is also developed, which is simple and efficient. Although initially developed for simple tolerance chains, interrelated tolerance chains can also be handled with the new methodology.

4. A decision aid for the interactive (indirect) interpretation of tolerances is also under development. The tolerance specification, optimization, analysis are achieved using several

modules, as outlined before. Similar modules are created for tolerance/finish compatibility establishment, operation selection and parameter selection, path generation and verification. The interpretation of output from each module is under investigation, at the present time. Such an interpretation is required for proper integration of all modules. all the programs are written in a personal computer environment (Zenith 386SX) and use several commercial software. The system is limited to simple part types at the present time but has been maintained sufficiently flexible to incorporate additional details.

5. It is believed that the prototype research and developmental tools created in this research will speed up the transfer of existing knowledge from research institutions to the manufacturing industry.

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